# Cryptography Engineering

- Lecture 4 (Nov 13, 2024)

- Today’s notes:
  - Secure Messaging
  - X3DH Protocol
  - Symmetric-key Ratchet

- Today’s coding tasks (and homework):
  - Implement X3DH using sockets

# Cryptography Engineering

- First Part of this Course: Key Exchange, Signature, and Handshake
  - Diffie-Hellman Key Exchange, and MitM attacks
  - Digital Signature and Certificate
  - Handshake Protocol
  - We addressed *How to Build a Secure Channel* over an open network…
    - *e.g., share a secure key, …*

- Second Part:
  - How to communicate securely over an open network…
  - Main Topic: Secure Messaging

# Secure Messaging

- Text Messages/Instant Messaging

WhatsApp

Signal

iMessage

# End-to-End Encryption

- End-to-End Encryption (E2EE)
  - Only sender and recipient can decrypt messages...
  - **The server cannot decrypt messages** (if it does not tamper with the conversation...
  - Confidentiality and Privacy
  - In practice, the server will help relaying/forwarding messages...

# End-to-End Encryption

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  - Confidentiality and Privacy
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E2EE (by default)
Examples

WhatsApp Signal iMessage Element

Non-E2EE (by default)
Examples

Discord Facebook Messager Telegram

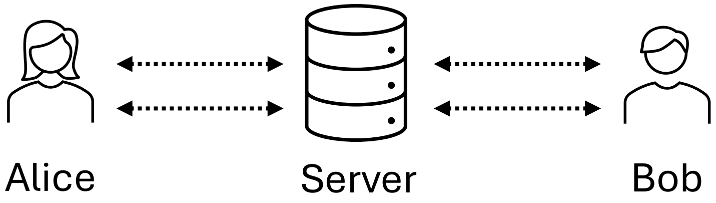
# End-to-End Encryption

**Initialization**

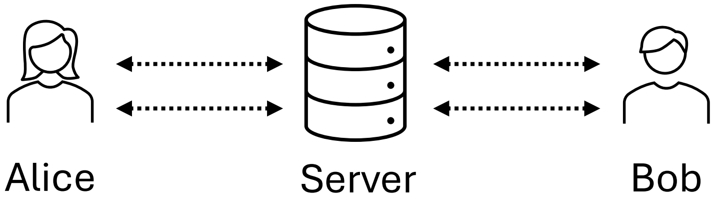
Including logging in, sharing users' information, …

**Messaging**

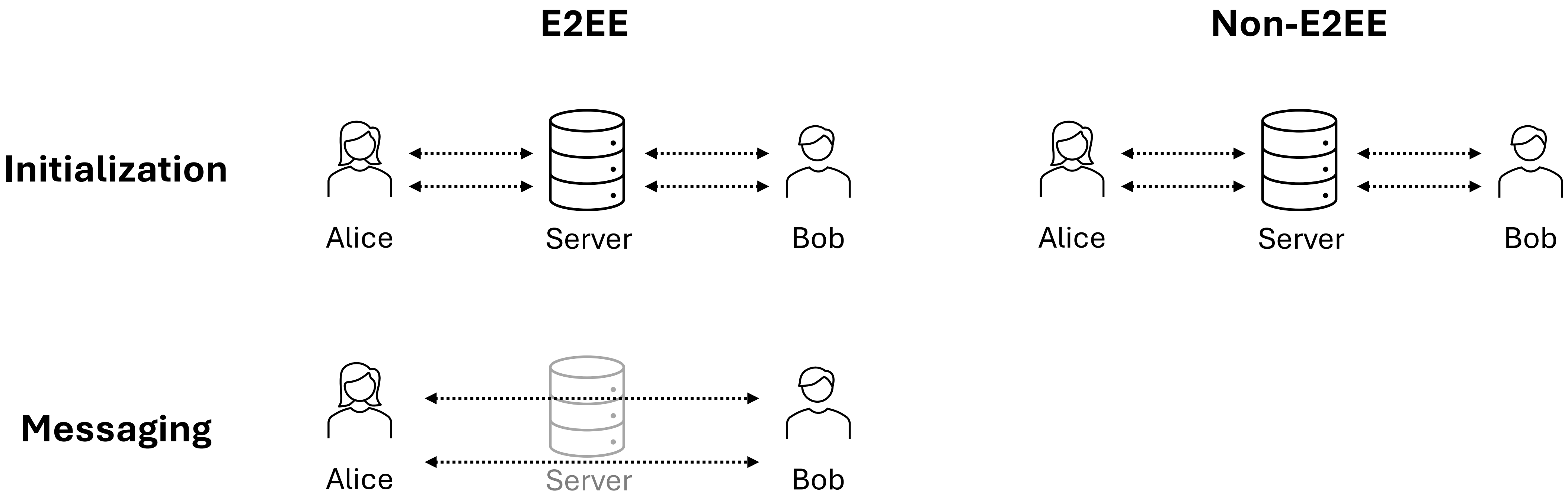
**E2EE**



**Non-E2EE**



# End-to-End Encryption



E2EE

Non-E2EE

Initialization

Alice Server Bob

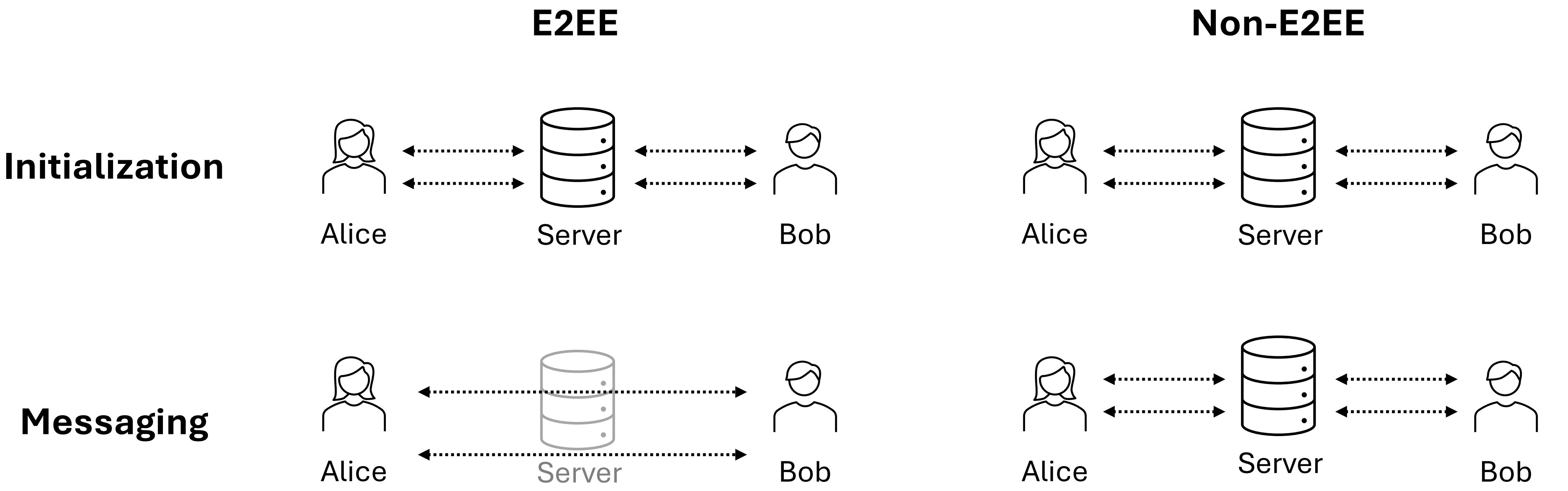
Alice Server Bob

Messaging

Alice Server Bob

The server **only relays** the encrypted messages, no storage ( or just short-term storage).

# End-to-End Encryption



E2EE

Non-E2EE

Initialization

Alice

Server

Bob

Alice

Server

Bob

Messaging

Alice

Server

Bob

Alice

Server

Bob

The server **only relays** the encrypted messages, no storage ( or just short-term storage).

Encrypted communication (e.g., via TLS) with the server, **but the messages may be stored (in plaintext)…**

# Signal Secure Messaging Protocol

Signal

Hey Now

Hey Now

How's it going? Now

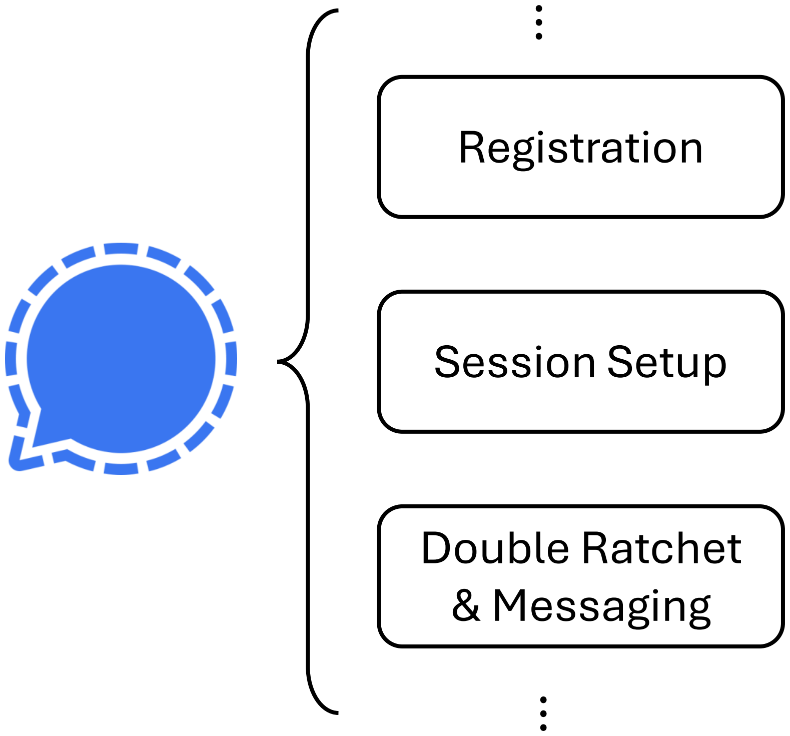
Good

How are you? Now

Fine Now

- One of the most secure instant messaging app
- End-to-end encryption (E2EE)
- WhatsApp also uses the Signal protocol

# Signal Secure Messaging Protocol



⋮

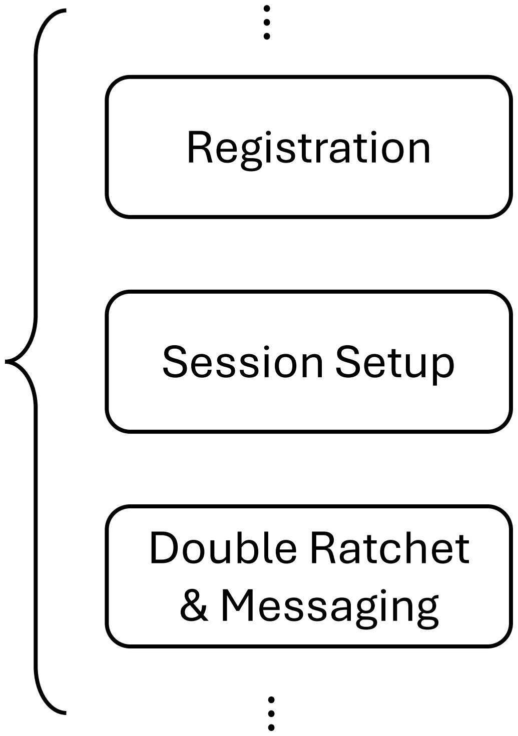
Registration

Session Setup

Double Ratchet & Messaging

⋮

# Signal Secure Messaging Protocol



⋮

Registration

Session Setup

Double Ratchet & Messaging

⋮

- Identity keys, signed pre-keys, one-time pre-keys, …
- X3DH (Extended Triple Diffie-Hellman) protocol **(Today)**
- Double Ratchet Algorithm:
    - Symmetric Ratchet **(Today)**
    - Diffie-Hellman Ratchet

# The X3DH Protocol

- Address *How to Establish Secure Initial Shared Secret*
  - It needs the server to help sharing pre-information
- Based on (EC)DH
- Mutual Authentication:
  - Two communication parties have long-term key pairs
- Provide Forward Secrecy

# The X3DH Protocol

- Key pairs of each party:
  - For simplicity, we define ‘$XPK$’ always equals to ‘$g^{xk}$’
  - All public keys (along with the user identity) will be stored in the server

Public parameters: $(\mathbb{G}, g, q)$:

A $q$-order EC group $\mathbb{G}$ with a generator $g$

| | Alice | Bob |
|---|---|---|
| Identity secret key (IK) | $ik_{\mathrm{A}} \in_{\$} \mathbb{Z}_q$ | $ik_{\mathrm{B}} \in_{\$} \mathbb{Z}_q$ |
| Identity public key (IPK) | $IPK_{\mathrm{A}}(= g^{ik_A})$ | $IPK_{\mathrm{B}}$ |
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**Identity keys**

- Generated during registration
- Will be used for **Key Exchange and Signing**

Public parameters: $(\mathbb{G}, g, q)$:

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**Signing Pre-keys**

- Generated during registration
- Updated periodically (e.g., once a week, or once a month)
- Will be used for **Key Exchange and Signing**

# The X3DH Protocol

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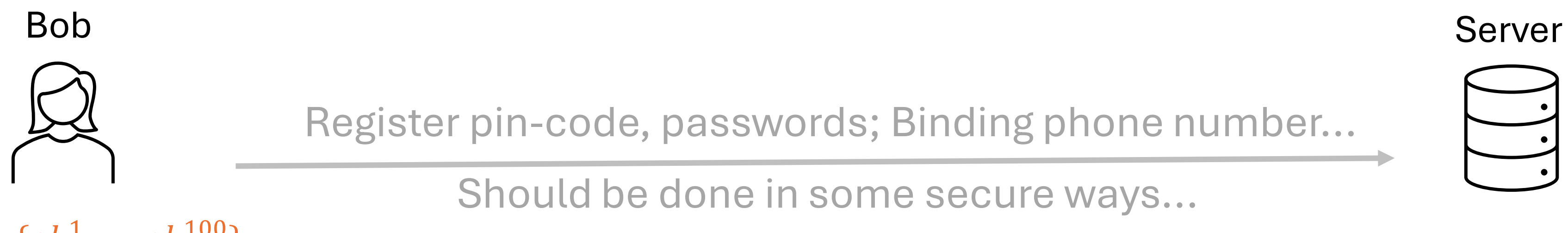
**One-time Pre-keys**

- Generated as a batch during registration
- Each key is used once for each new session; Deleted after use
- Re-generated when used up (or the supply is low)

# The X3DH Protocol

- When Bob registers (we only focus on the cryptographic parts)…
  - For simplicity, we define $‘XPK’$ always equals to $‘g^{xk}’$



Bob

Server

Register pin-code, passwords; Binding phone number…

Should be done in some secure ways…

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

**Prekey bundle database**

**Alice:** $IPK_\mathrm{A}, SPK_A, \sigma_A, \{OPK_A^1, \dots, OPK_A^{100}\}$**, …**

**…**

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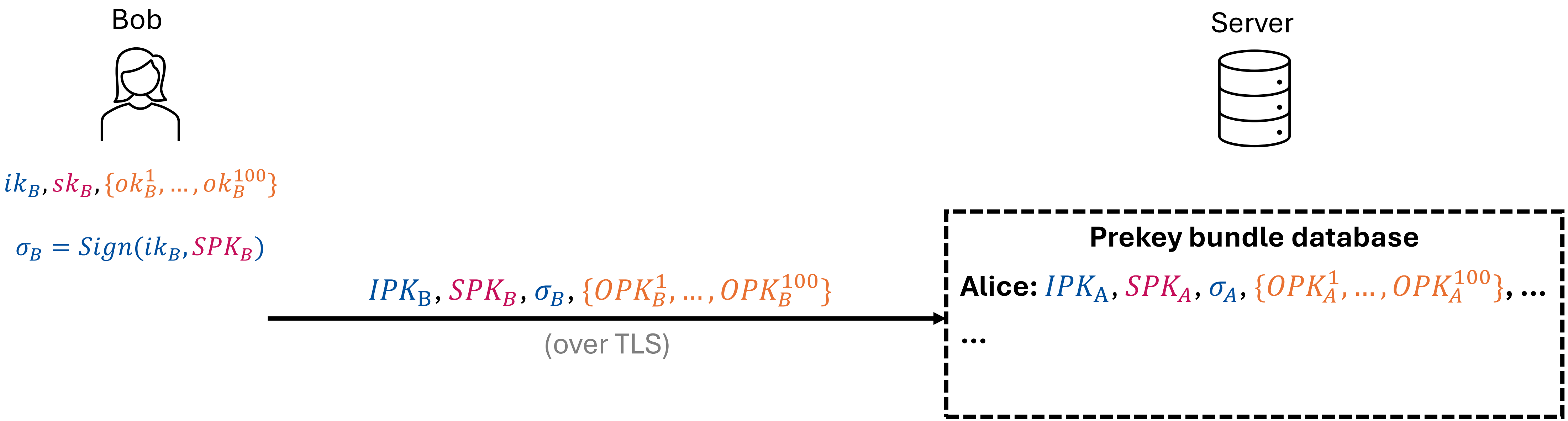

Bob

Server

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

$\sigma_B = Sign(ik_B, SPK_B)$

$IPK_\mathrm{B}, SPK_B, \sigma_B, \{OPK_B^1, \dots, OPK_B^{100}\}$

(over TLS)

**Prekey bundle database**

**Alice:** $IPK_\mathrm{A}, SPK_A, \sigma_A, \{OPK_A^1, \dots, OPK_A^{100}\}$**, …**

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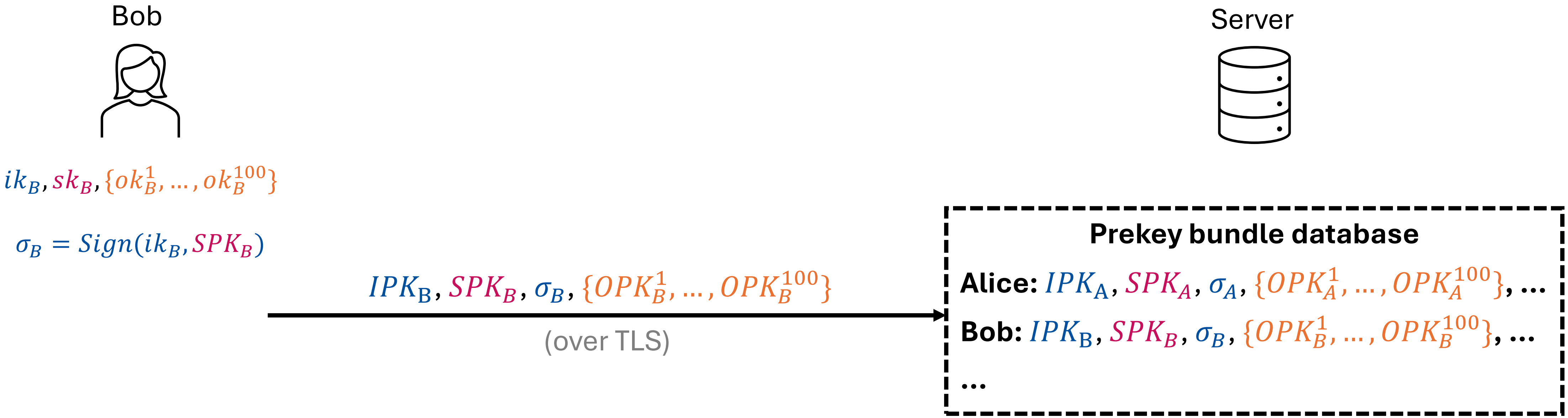

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$\sigma_B = Sign(ik_B, SPK_B)$

$$\xrightarrow[\text{(over TLS)}]{IPK_{\mathrm{B}}, SPK_B, \sigma_B, \{OPK_B^1, \ldots, OPK_B^{100}\}}$$

**Prekey bundle database**

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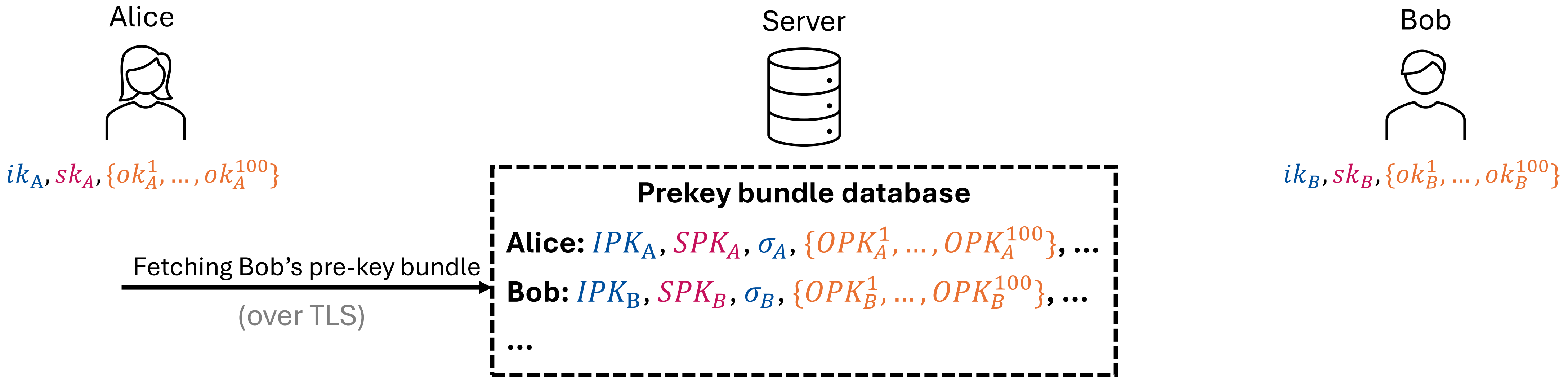
**Bob:** $IPK_{\mathrm{B}}, SPK_B, \sigma_B, \{OPK_B^1, \ldots, OPK_B^{100}\}$**, …**

**…**

# The X3DH Protocol

- When Alice communicates with Bob…



Alice

Server

Bob

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

**Prekey bundle database**

**Alice:** $IPK_A, SPK_A, \sigma_A, \{OPK_A^1, \dots, OPK_A^{100}\}$, …

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…

Fetching Bob’s pre-key bundle

(over TLS)

# The X3DH Protocol

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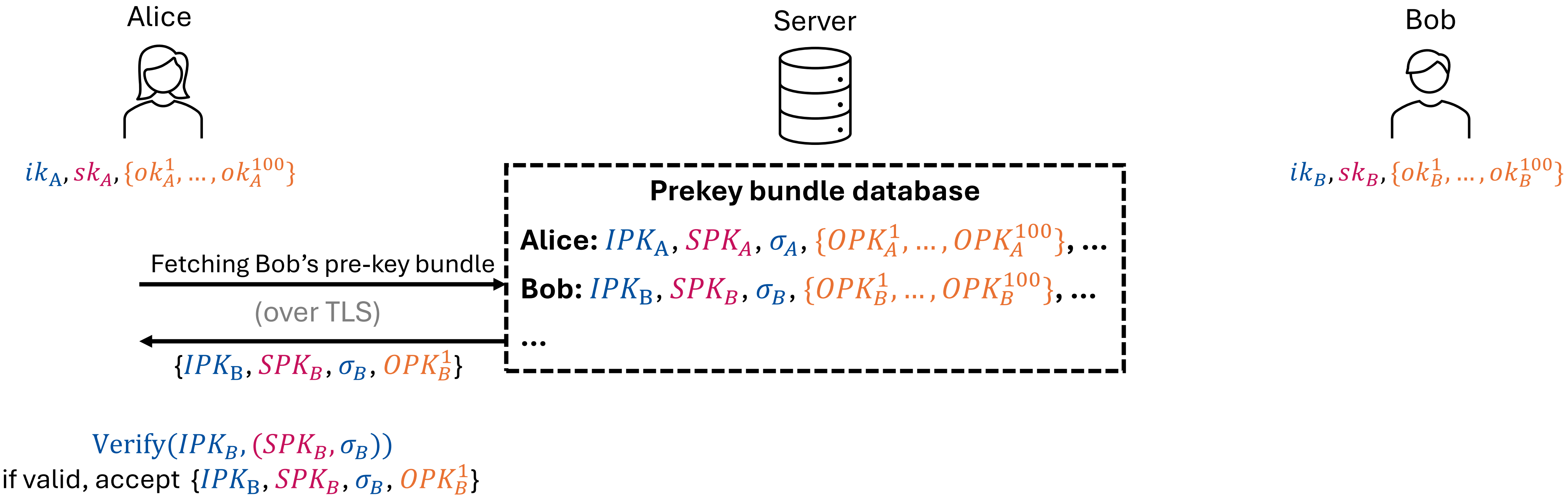

Alice

Server

Bob

$ik_{\mathrm{A}}, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

**Prekey bundle database**

**Alice:** $IPK_{\mathrm{A}}, SPK_A, \sigma_A, \{OPK_A^1, \dots, OPK_A^{100}\}, \dots$

**Bob:** $IPK_{\mathrm{B}}, SPK_B, \sigma_B, \{OPK_B^1, \dots, OPK_B^{100}\}, \dots$

**...**

Fetching Bob’s pre-key bundle

(over TLS)

$\{IPK_{\mathrm{B}}, SPK_B, \sigma_B, OPK_B^1\}$

$\mathrm{Verify}(IPK_B, (SPK_B, \sigma_B))$

if valid, accept $\{IPK_{\mathrm{B}}, SPK_B, \sigma_B, OPK_B^1\}$

# The X3DH Protocol

- When Alice communicates with Bob...

Alice

$ik_{\mathrm{A}}, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$\{IPK_{\mathrm{B}}, SPK_B, OPK_B^1\}$

Bob

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

# The X3DH Protocol

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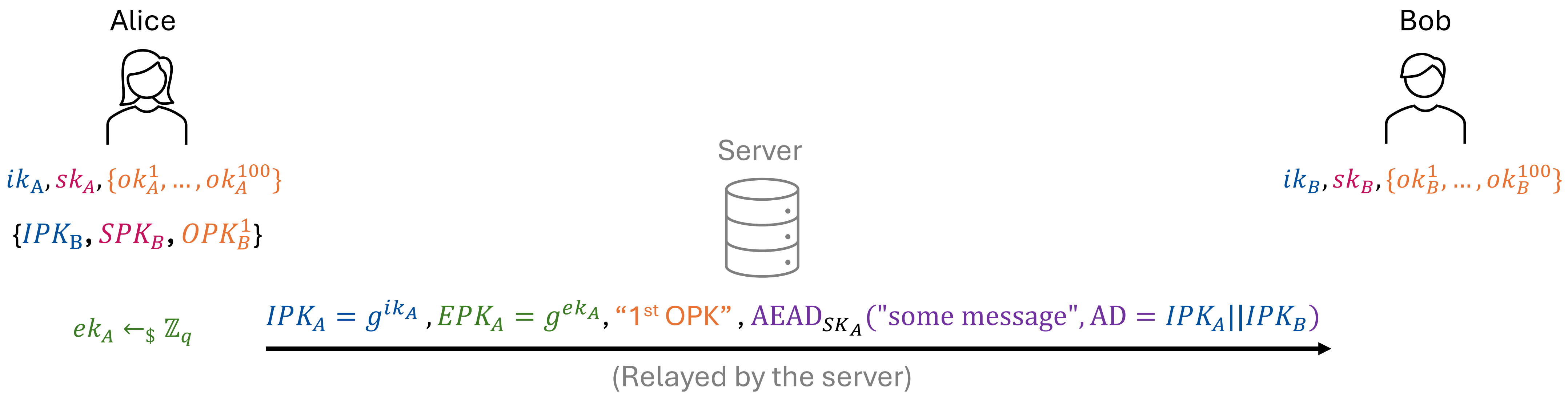


Alice

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$\{IPK_B, SPK_B, OPK_B^1\}$

$ek_A \leftarrow_\$ \mathbb{Z}_q$

Server

Bob

$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

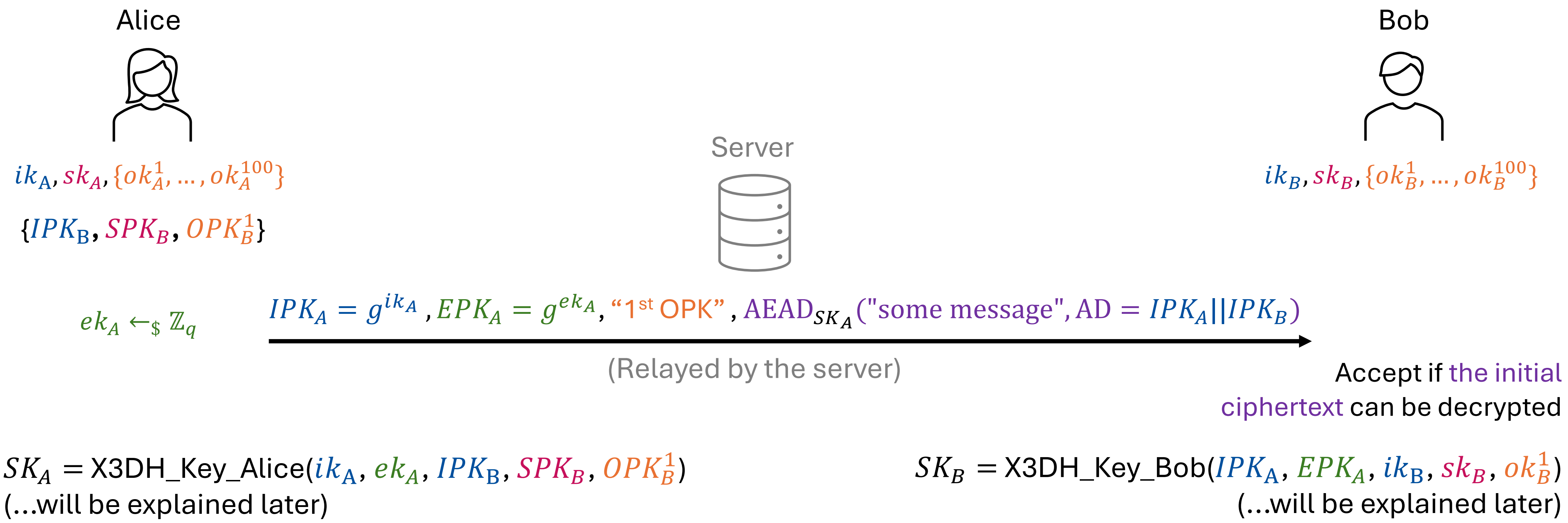
$IPK_A = g^{ik_A}, EPK_A = g^{ek_A}$, "1st OPK", $\text{AEAD}_{SK_A}(\text{"some message"}, \text{AD} = IPK_A || IPK_B)$

(Relayed by the server)

$SK_A$ = X3DH_Key_Alice($ik_A, ek_A, IPK_B, SPK_B, OPK_B^1$)
(…will be explained later)

# The X3DH Protocol

- When Bob receives messages (which is actually relayed by the server) from Alice...



Alice

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$\{IPK_B, SPK_B, OPK_B^1\}$

Server

Bob

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(Relayed by the server)

Accept if the initial ciphertext can be decrypted

$SK_A =$ X3DH_Key_Alice($ik_A, ek_A, IPK_B, SPK_B, OPK_B^1$)
(...will be explained later)

$SK_B =$ X3DH_Key_Bob($IPK_A, EPK_A, ik_B, sk_B, ok_B^1$)
(...will be explained later)

# The X3DH Protocol

- How the X3DH protocol computes a shared secret...

Alice

$SK_A = \text{X3DH\_Key\_Alice}(ik_{\mathrm{A}}, ek_A, IPK_{\mathrm{B}}, SPK_B, OPK_B)$

1. $\mathrm{DH}_1 = SPK_B^{ik_A}$
2. $\mathrm{DH}_2 = IPK_{\mathrm{B}}^{ek_A}$
3. $\mathrm{DH}_3 = SPK_B^{ek_A}$
4. $\mathrm{DH}_4 = (OPK_B)^{ek_A}$
5. $SK_{\mathrm{A}} = \mathrm{KDF}(\mathrm{DH}_1, \mathrm{DH}_2, \mathrm{DH}_3, \mathrm{DH}_4)$

Bob

$SK_B = \text{X3DH\_Key\_Bob}(IPK_{\mathrm{A}}, EPK_A, ik_{\mathrm{B}}, sk_B, ok_B)$

1. $\mathrm{DH}_1 = IPK_A^{sk_B}$
2. $\mathrm{DH}_2 = EPK_A^{ik_B}$
3. $\mathrm{DH}_3 = EPK_A^{sk_B}$
4. $\mathrm{DH}_4 = EPK_A^{ok_B}$
5. $SK_{\mathrm{B}} = \mathrm{KDF}(\mathrm{DH}_1, \mathrm{DH}_2, \mathrm{DH}_3, \mathrm{DH}_4)$

# The X3DH Protocol

- How the X3DH protocol computes a shared secret...

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5. $SK_{\mathrm{A}} = \mathrm{KDF}(\mathrm{DH}_1, \mathrm{DH}_2, \mathrm{DH}_3, \mathrm{DH}_4)$

Bob

$SK_B = \text{X3DH\_Key\_Bob}(IPK_{\mathrm{A}}, EPK_A, ik_{\mathrm{B}}, sk_B, ok_B)$

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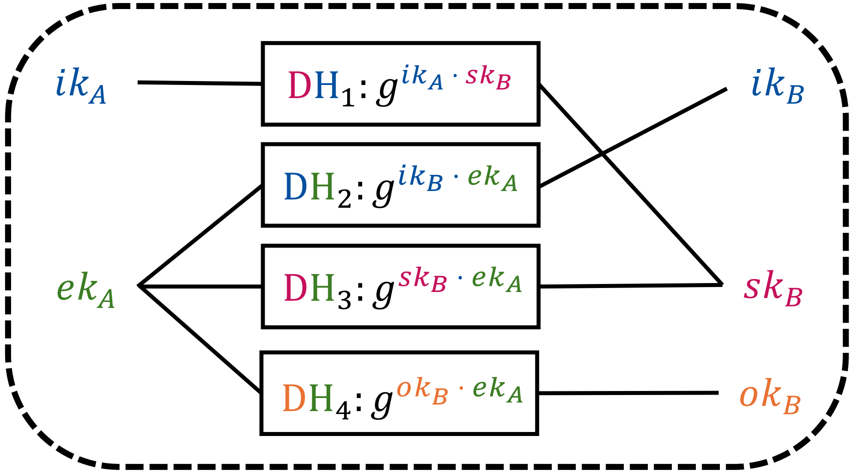


$ik_A$

$ek_A$

$\mathrm{DH}_1\colon g^{ik_A \cdot sk_B}$

$\mathrm{DH}_2\colon g^{ik_B \cdot ek_A}$

$\mathrm{DH}_3\colon g^{sk_B \cdot ek_A}$

$\mathrm{DH}_4\colon g^{ok_B \cdot ek_A}$

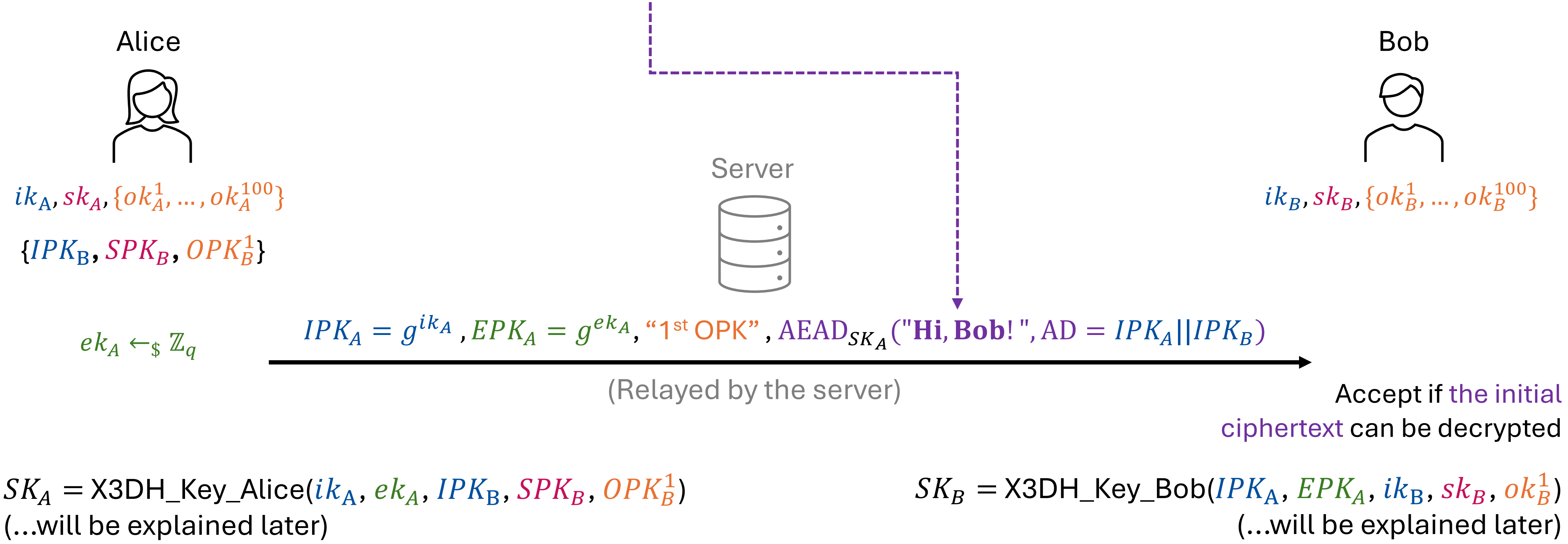
$ik_B$

$sk_B$

$ok_B$

# The X3DH Protocol

- **0-RTT** (Zero Round-Trip Time): Send message instantly without waiting response



Alice

Server

Bob

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

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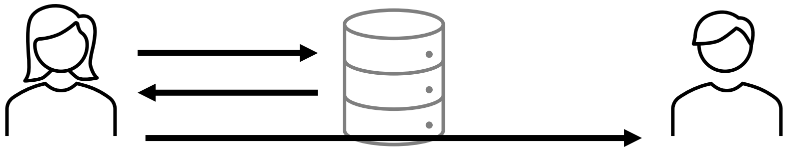
(...will be explained later)

# The X3DH Protocol

- Based on (EC)DH
- Trusted server required
  - Store public keys, relay messages, ...
  - Cannot decrypt ciphertexts...
- 0-RTT
  - Immediate message sending without waiting for a response
- Support offline communication
  - Can be executed even if Bob (the receiver) is offline
  - Offline messages (encrypted) will be stored in the server until Bob is online again
- Mutual Authentication, Forward Secrecy, ...
  - In this Course, we focus on *How it works* rather than *Why it is secure*...



X3DH

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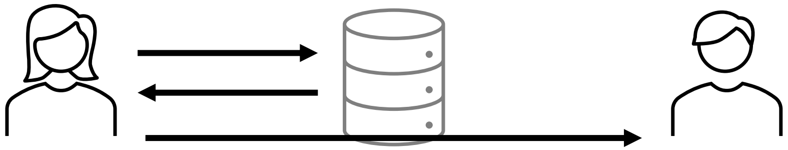


X3DH

**A note: Do not confuse X3DH with TLS**

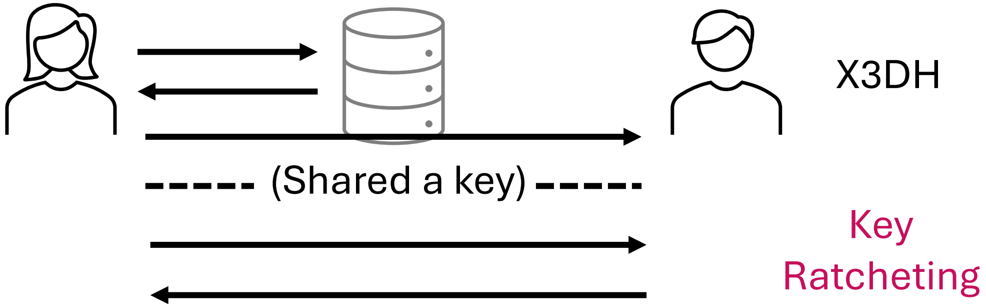
Different primary goals/settings:

X3DH: secure messaging between users, rely on trusted pre-shared public keys...

TLS: secure connections with a server, rely on trusted CAs and use certificates...

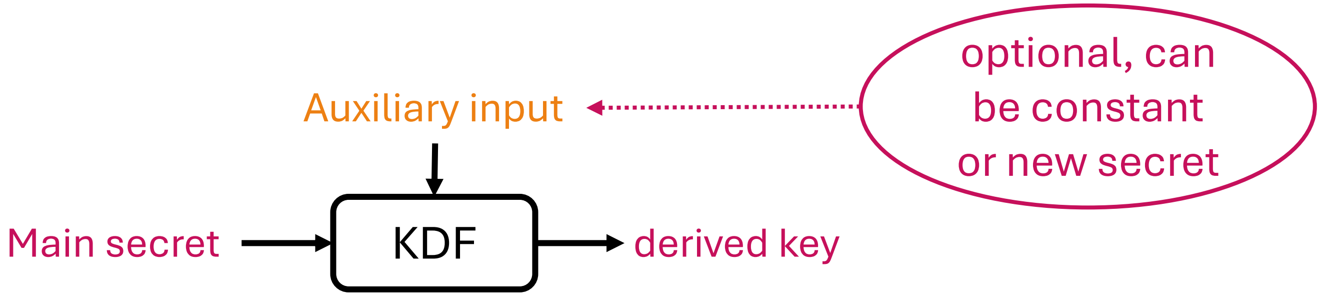
# Symmetric Key Ratchet

- After completing X3DH…
- … we use **Double Ratchet** to:
  - Encrypt messages + updates the shared key
  - ~~Encrypt messages using the same shared key~~
  - Diffie-Hellman Ratchet + Symmetric-key Ratchet
- Essential for forward/backward secrecy (next lecture)
- Today: Symmetric Key Ratchet



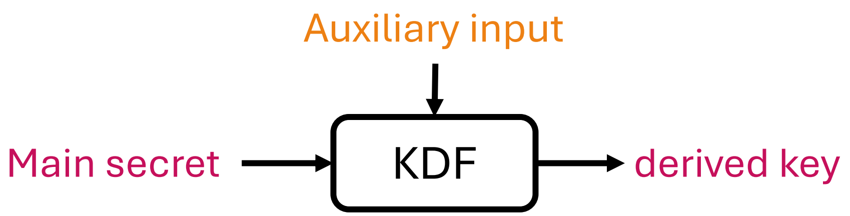
# Symmetric Key Ratchet

- KDF chain
    - KDF: Key derivation function

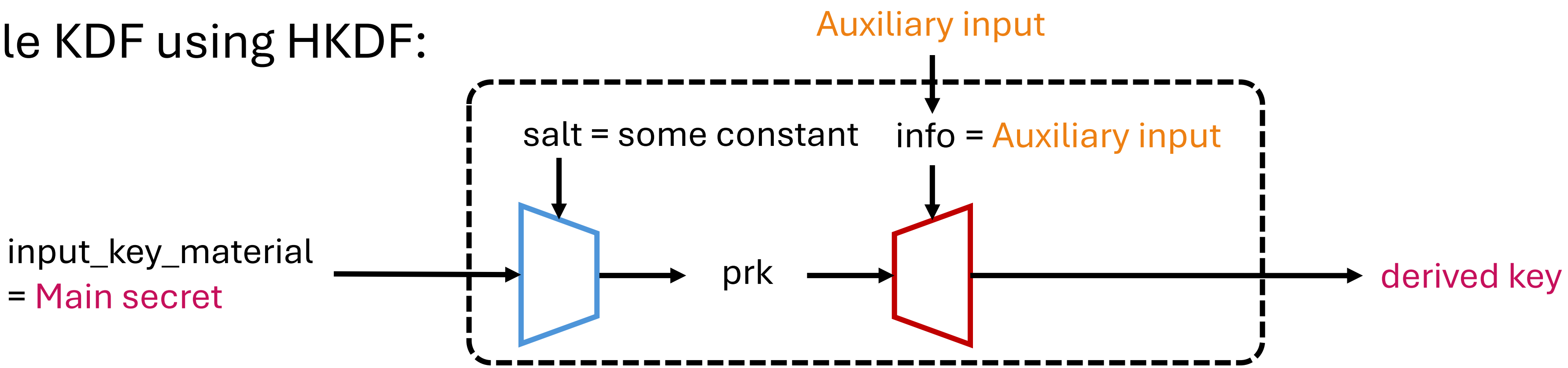


# Symmetric Key Ratchet

- KDF chain
  - KDF: Key derivation function



- Example KDF using HKDF:

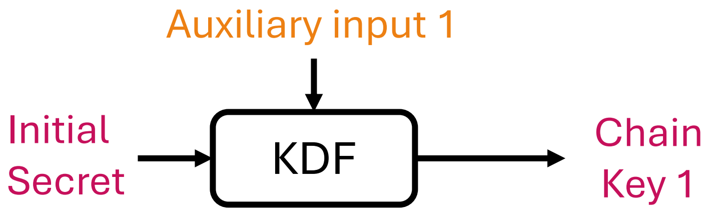


1. prk = HKDF.**Extract**( input_key_material = Main secret, salt = some constant )

2. derived key = HKDF.**Expand**( prk, Auxiliary input)

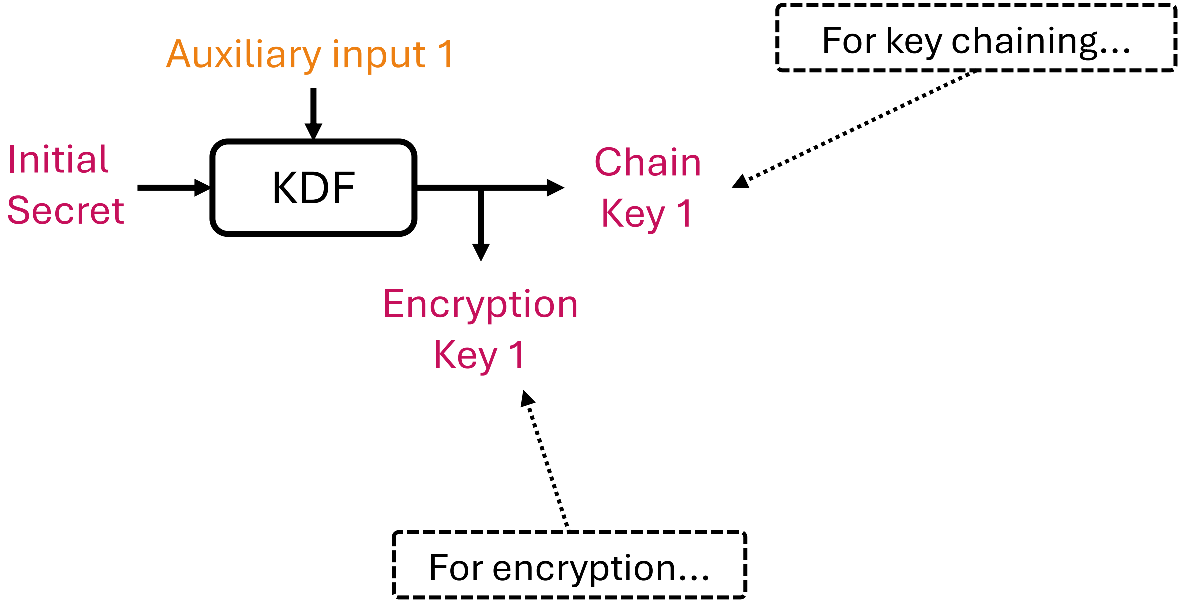
# Symmetric Key Ratchet

- KDF chain
  - KDF: Key derivation function



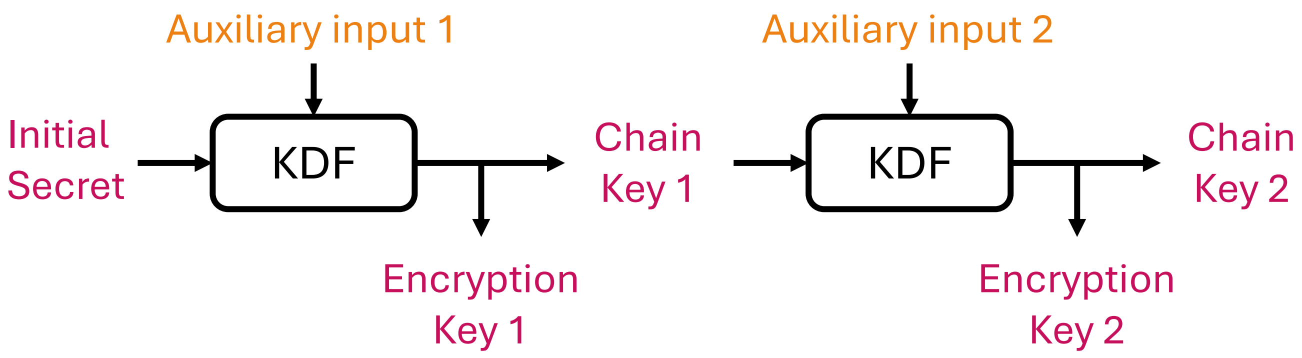
# Symmetric Key Ratchet

- KDF chain
  - KDF: Key derivation function



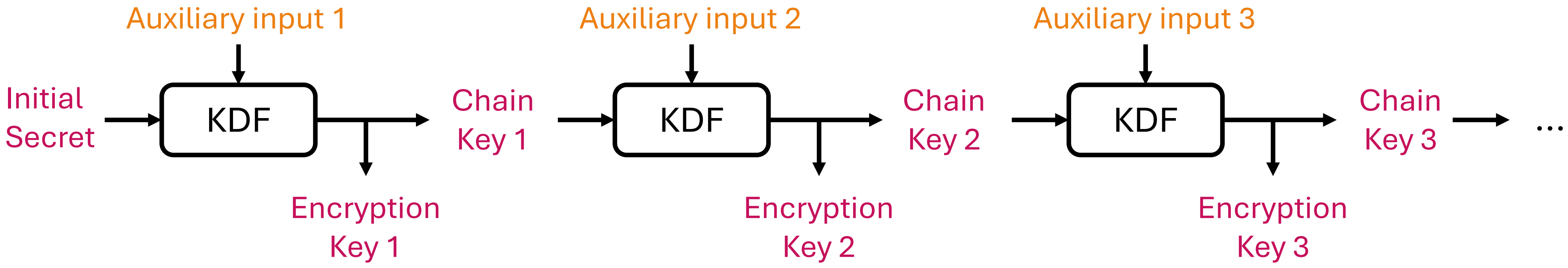
# Symmetric Key Ratchet

- KDF chain
  - KDF: Key derivation function



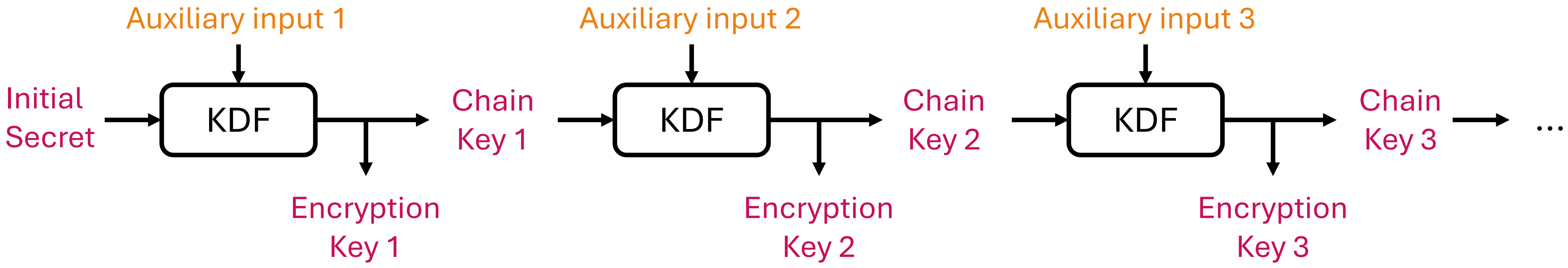
# Symmetric Key Ratchet

- KDF chain
  - KDF: Key derivation function



# Symmetric Key Ratchet

- KDF chain
    - KDF: Key derivation function



- Use Key Chain to encrypt messages (next lecture)

# Coding Tasks

1. Implement a KDF chain based on HKDF.
    - You can learn how to use HKDF in the example code “HKDF.py” of Lecture 3.
    - To split a KDF output into Encryption Key and Chain Key, you can first specify the “length” parameter of hkdf_expand, and then truncate it into two byte-strings.

# Homework

- **Homework:** Try implementing X3DH using sockets:

  1. Suppose that Alice and Bob have registered with the server. Namely, the server has stored prekey bundles of Alice and Bob.

  2. Alice wants to communicate with Bob, it first fetches prekey bundle of Bob from the server.

  3. Upon receiving the prekey bundle of Bob, Alice verifies the bundle. If it is valid, then Alice follows the X3DH protocol and compute a shared key. After computing a shared key, it sends the protocol message (see the X3DH protocol in this lecture note) to the server.

  4. The server forwards the message from Alice to Bob.

  5. Upon receiving the message from Alice, Bob also compute the X3DH session key.

- **Bonus:** Upgrade your implementation of X3DH so that it allows the recipient user to be offline.

# Further Reading

- Old news -- *WhatsApp's Signal Protocol integration is now complete: https://signal.org/blog/whatsapp-complete/*
- Technical Documentations of Signal: https://signal.org/docs/
- Cohn-Gordon et al’s security analysis of Signal: https://eprint.iacr.org/2016/1013