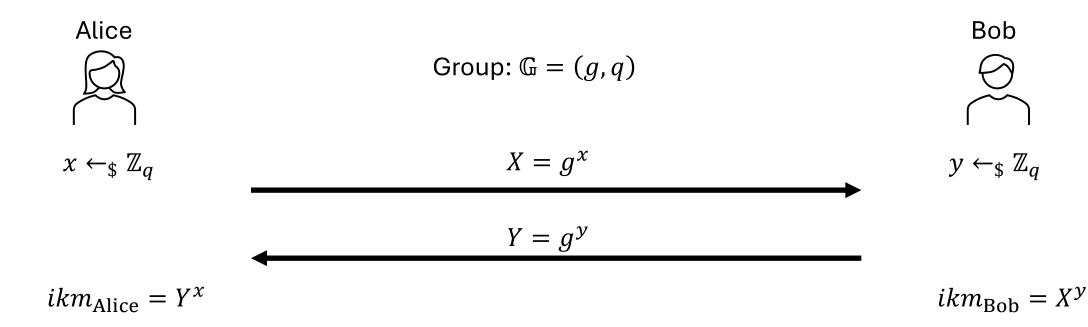
# **Cryptography Engineering**

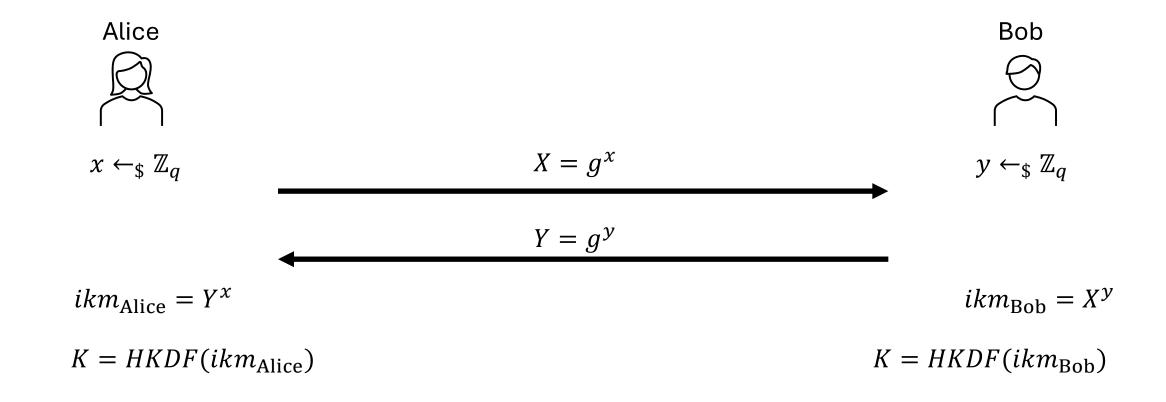
- Lecture 2 (Oct 29, 2025)
- Today's notes:
  - Review the Rust example code
  - DH handshake
  - Man-in-the-middle attacks
- Today's coding tasks:
  - Derive a secret key for AEAD via DH handshake
  - Man-in-the-Middle attacks on DHKE

#### **Code Review**

- Some useful notes:
  - Modular design
  - Reusability

- Group (Mathematics): (G, +)
  - Associativity:  $a, b, c \in \mathbb{G} \implies (a+b) + c = a + (b+c)$
  - Identity:  $\exists e \in \mathbb{G}$  s.t.  $\forall a \in \mathbb{G} \implies e + a = a$
  - Inverse:  $\forall a \in \mathbb{G}$ ,  $\exists b \in \mathbb{G}$  s.t. a + b = e
  - Example 1:  $(\mathbb{R}, +)$  is a group, but  $(\mathbb{Z}, \times)$  is not
  - Example 2:  $(\mathbb{R}, \times)$  is **not** a group (why?), but  $(\mathbb{R}^*, \times)$  is a group
  - Quick question:  $(\{1,2,3,...,q\}, \times mod q)$  is a group if and only if q is \_\_\_\_\_
- In cryptography, we usually use finite groups to build cryptosystems.
  - Generator and order
  - Elliptic Curve Groups





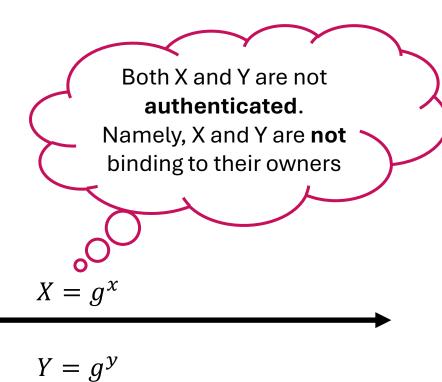


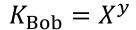
• Diffie-Hellman Key Exchange

Alice

$$x \leftarrow_{\$} \mathbb{Z}_q$$

 $K_{\text{Alice}} = Y^x$ 





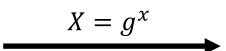
Bob

 $y \leftarrow_{\$} \mathbb{Z}_q$ 

• Diffie-Hellman Key Exchange



$$x \leftarrow_{\$} \mathbb{Z}_q$$



Adversary





• Diffie-Hellman Key Exchange



$$X = g^x$$

Adversary



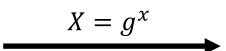




• Diffie-Hellman Key Exchange



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Adversary

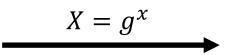




• Diffie-Hellman Key Exchange



$$x \leftarrow_{\$} \mathbb{Z}_q$$



Adversary



 $X'=g^{x\prime}$ 

$$x' \leftarrow_{\$} \mathbb{Z}_q$$



• Diffie-Hellman Key Exchange



$$X = g^x$$

Adversary



$$x' \leftarrow_{\$} \mathbb{Z}_q$$

 $X'=g^{x\prime}$ 



$$y \leftarrow_{\$} \mathbb{Z}_q$$

$$K_{\text{Bob}} = X'^{y}$$

• Diffie-Hellman Key Exchange



$$x \leftarrow_{\$} \mathbb{Z}_q$$

$$X = g^x$$

Adversary



$$x' \leftarrow_{\$} \mathbb{Z}_q$$

$$X'=g^{x\prime}$$

$$Y=g^{y}$$



$$y \leftarrow_{\$} \mathbb{Z}_q$$

$$K_{\text{Bob}} = X'^{y}$$

• Diffie-Hellman Key Exchange



$$x \leftarrow_{\$} \mathbb{Z}_q$$

$$X = g^x$$

$$K_{\text{Alice}} = Y'^{x}$$

Adversary



$$x' \leftarrow_{\$} \mathbb{Z}_c$$

$$x' \leftarrow_{\$} \mathbb{Z}_q$$
$$y' \leftarrow_{\$} \mathbb{Z}_q$$



$$X'=g^{x'}$$

$$X_{\mathbf{q}}$$
  $Y = g^{\mathcal{Y}}$ 



$$y \leftarrow_{\$} \mathbb{Z}_q$$

$$K_{\text{Bob}} = X'^{y}$$

Diffie-Hellman Key Exchange



$$X = g^x$$

$$K_{\text{Alice}} = Y'^{x}$$

Adversary



$$x' \leftarrow_{\$} \mathbb{Z}_q$$

$$y' \leftarrow_{\$} \mathbb{Z}_q$$

$$K'_{\text{Bob}} = Y^{x'}$$
 $K'_{\text{Alice}} = X^{y'}$ 

$$X'=g^{x\prime}$$

$$X'=g^{X'}$$

$$Y = g^{y}$$



$$y \leftarrow_{\$} \mathbb{Z}_q$$

$$K_{\text{Bob}} = X'^{y}$$

• Diffie-Hellman Key Exchange



$$x \leftarrow_{\$} \mathbb{Z}_q$$

 $K_{\text{Alice}} = Y'^{x}$ 

$$V' = a^{y'}$$

Communicate with  $K'_{Alice}$ 

 $X = g^x$ 

Adversary



$$x' \leftarrow_{\$} \mathbb{Z}_q$$

$$y' \leftarrow_{\$} \mathbb{Z}_q$$

$$K'_{\text{Bob}} = Y^{x'}$$
  
 $K'_{\text{Alice}} = X^{y'}$ 

Communicate with 
$$K'_{
m Bob}$$

 $X'=g^{x\prime}$ 

 $Y = g^{y}$ 

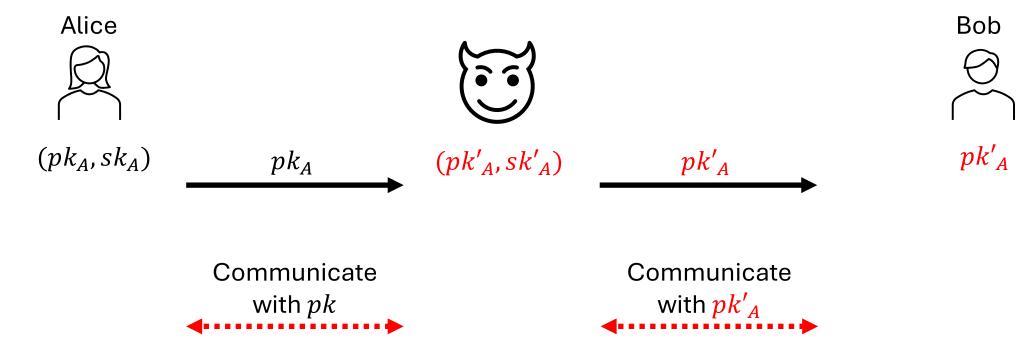


$$y \leftarrow_{\$} \mathbb{Z}_q$$

$$K_{\text{Bob}} = X'^{y}$$

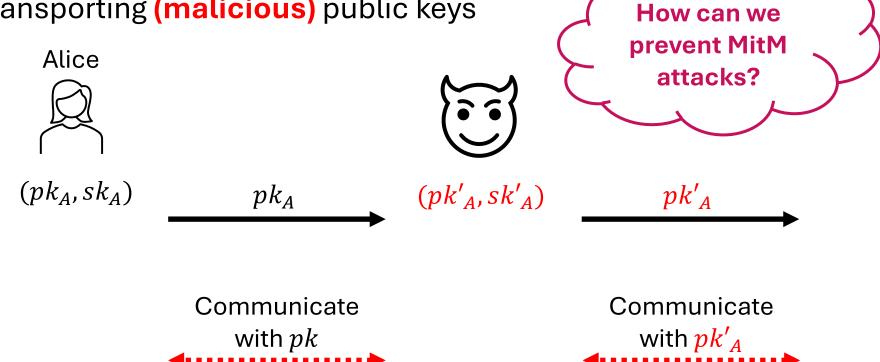
# MitM attacks (in General)

• Transporting (malicious) public keys



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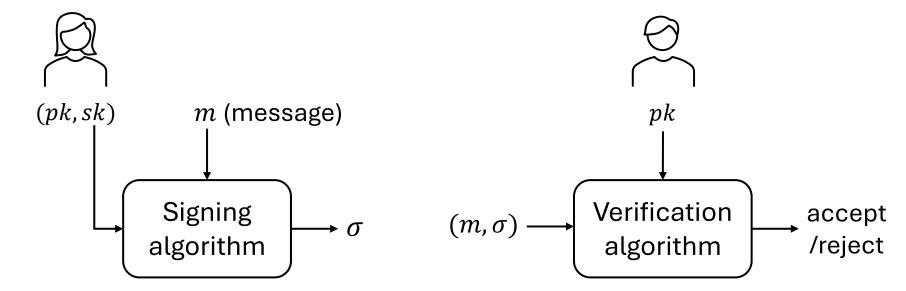


Bob

 $pk'_A$ 

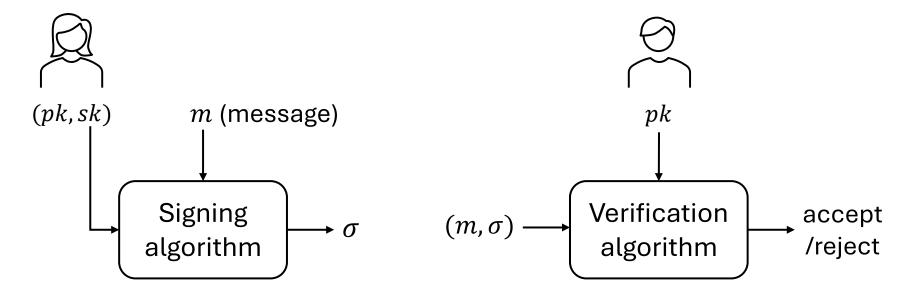
### **Digital Signature**

• Signature Schemes



### **Digital Signature**

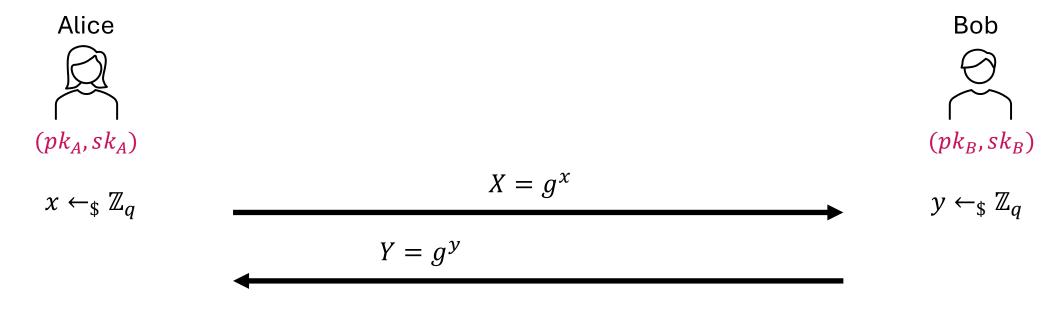
• Signature Schemes



- Security: Unforgeability
  - Unable to forge a valid signature on any message without sk

# Signed DH Key Exchange (Next Lecture)

• Use signature to avoid MitM attacks on DHKE:

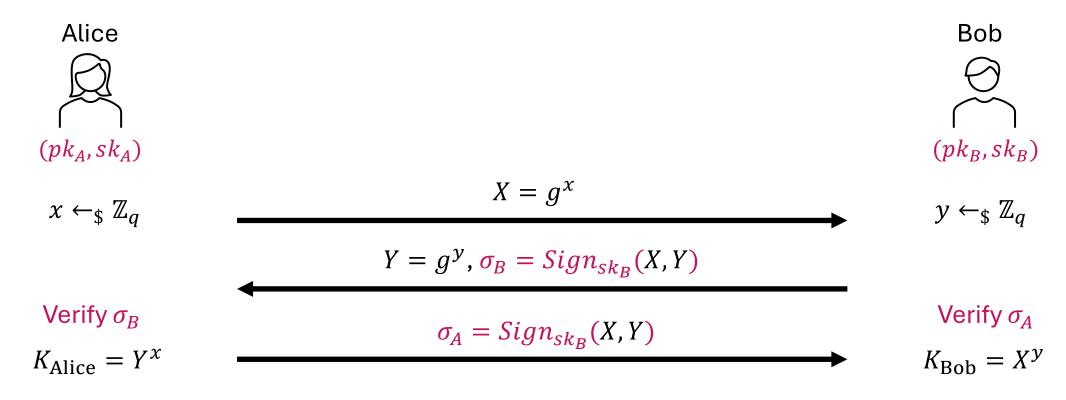


$$K_{\text{Alice}} = Y^{x}$$

$$K_{\text{Bob}} = X^{y}$$

# Signed DH Key Exchange (Next Lecture)

• Use signature to avoid MitM attacks on DHKE:



### **Digital Signature**

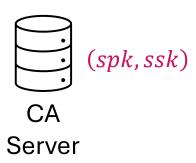
Other standard properties of Digital Signature:

```
    Authentication // Verify the identity...
    Publicly verifiable // Everyone with pk can verify the signature...
    Non-repudiation // A party cannot deny having sent or signed a message...
```

• One of the most important applications: Digital Certificate

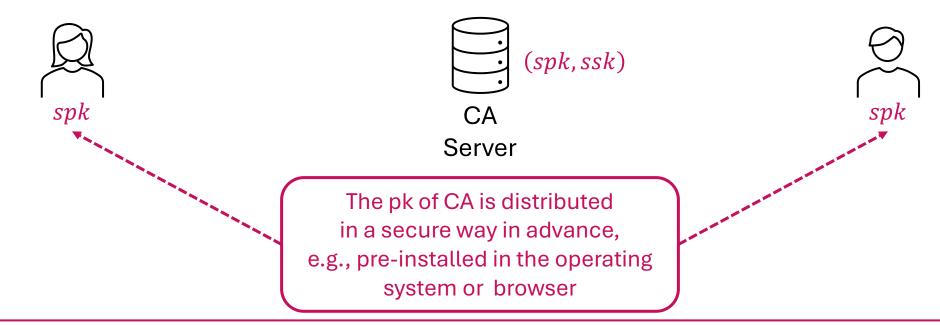
- Certificate: A signature generated by a trusted party (In short)
  - Verifies an ID and binds it to a public key
  - Securely distribute public keys
  - Issued by **CA** (Certificate Authority)



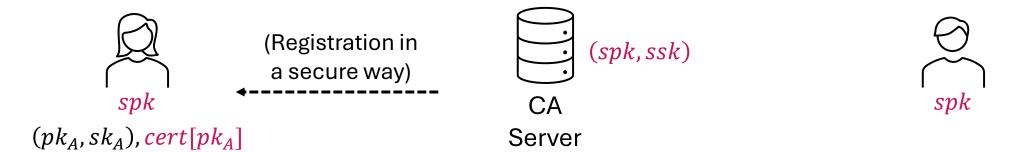




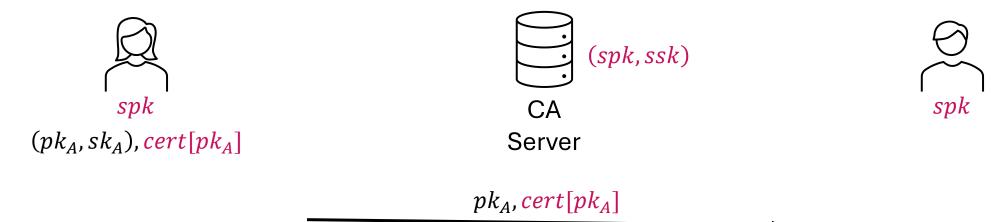
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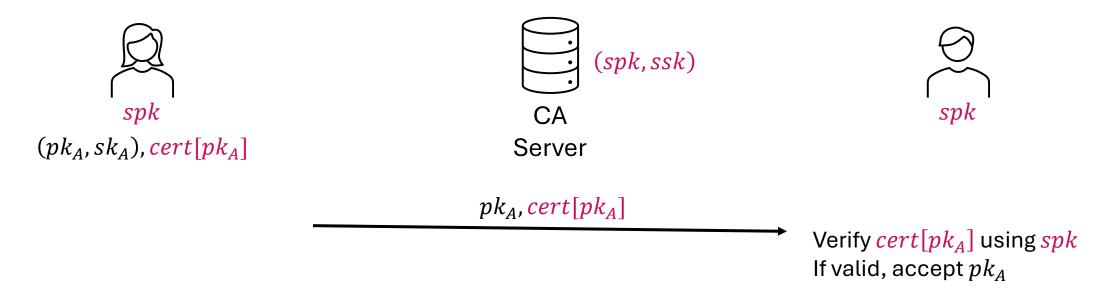
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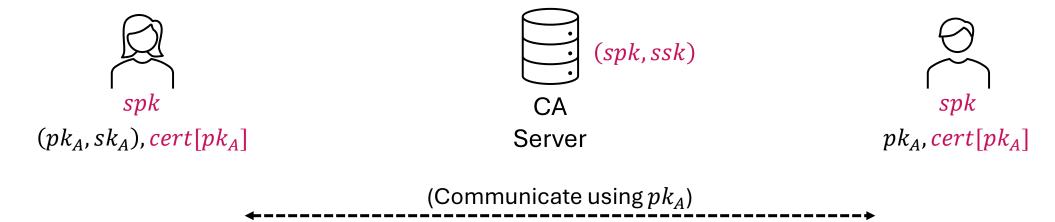
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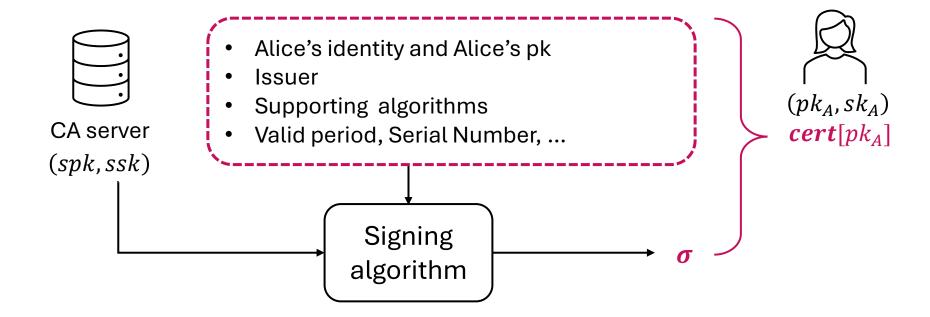
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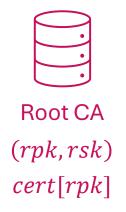


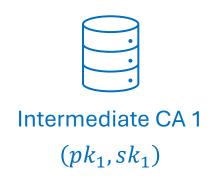
- What information does a certificate include?
  - X.509 standard: defines the format of public key certificates.



- Root Certificate and Certificate Chains
  - Hierarchical sequence of certificates
  - Trace the authenticity of a certificate back to a trusted Root CA
  - Only **root certificates** need to be pre-installed...

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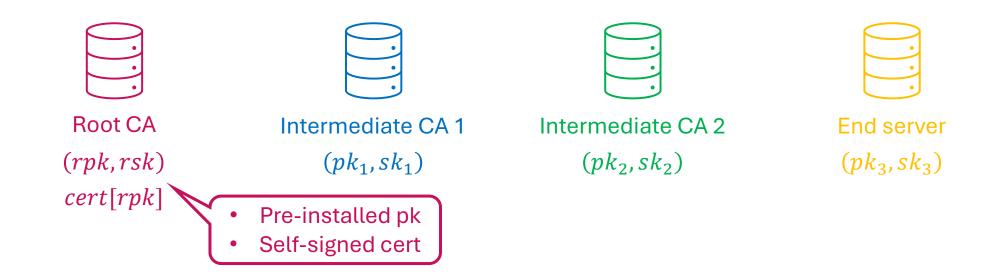




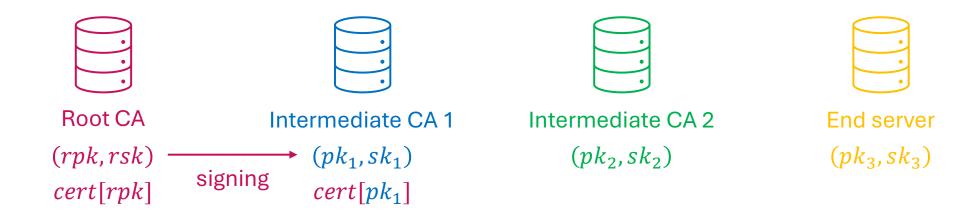




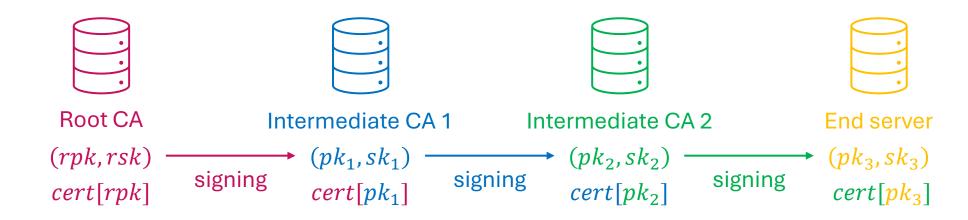
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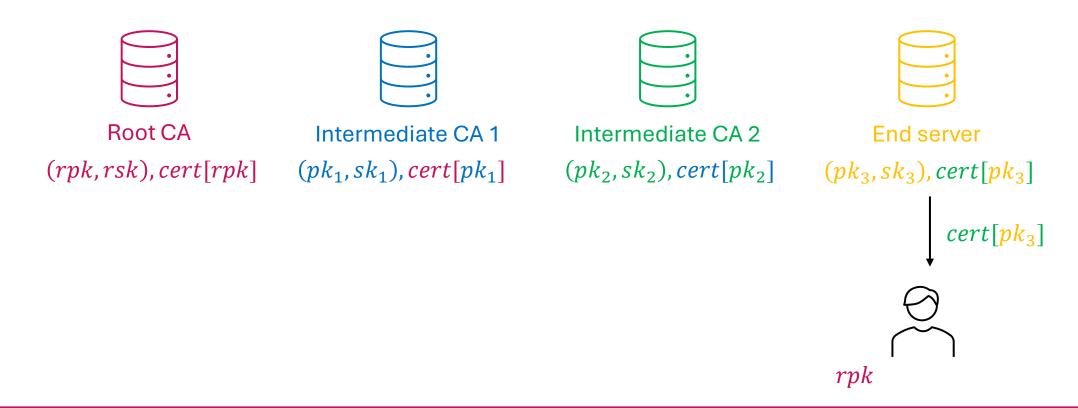


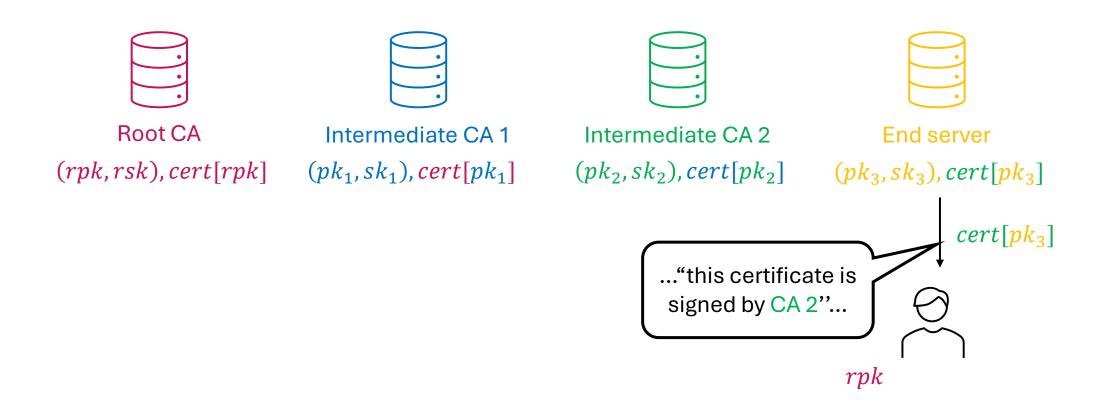
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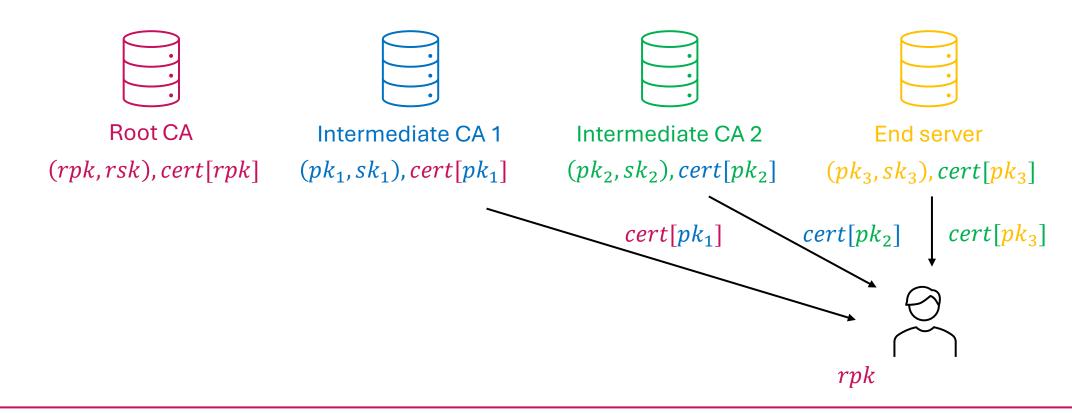


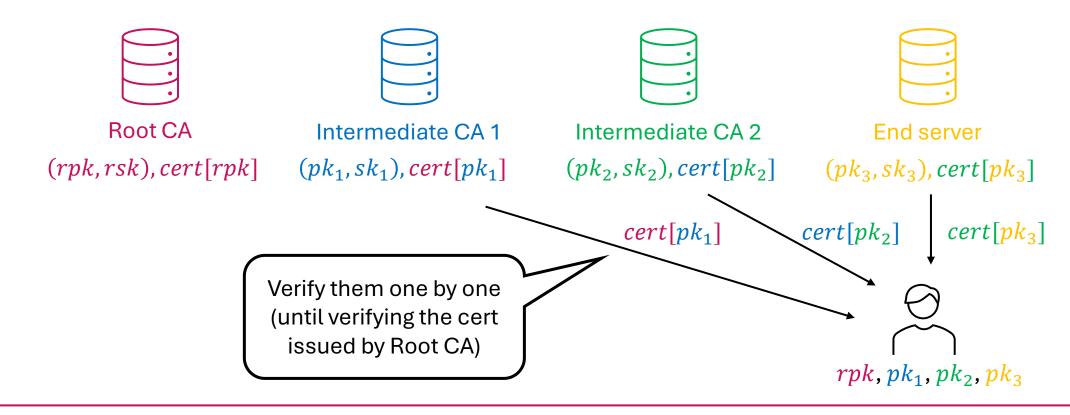
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  - Trace the authenticity of a certificate back to a trusted Root CA
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#### **Exercise**

- 1. Export a certificate from a website and write a simple program to read the certificate.
- 2. Find and export a pre-installed certificate on your laptop or PC (via browser) and use your program to read the certificate.
- 3. Implement the DH key exchange and derive a key using the shared DH secret.
- 4. Implement the man-in-the-middle attacks on the DH key exchange.

### **Further Reading**

- DigiCert (one of the largest and most widely trusted CAs): <a href="https://www.digicert.com/">https://www.digicert.com/</a>
- Elliptic Curves: <a href="https://andrea.corbellini.name/2015/05/17/elliptic-curve-cryptography-a-gentle-introduction/">https://andrea.corbellini.name/2015/05/17/elliptic-curve-cryptography-a-gentle-introduction/</a>
- P-256 (secp256r1) curve: https://neuromancer.sk/std/nist/P-256
- The X.509 standard: https://en.wikipedia.org/wiki/X.509
- Public Key Infrastructure (PKI): <a href="https://en.wikipedia.org/wiki/Public\_key\_infrastructure">https://en.wikipedia.org/wiki/Public\_key\_infrastructure</a>