Cryptography Engineering

- Lecture 7 (Dec 3, 2025)

- Case study: **E2EE-secure messaging** (2 lectures)
  - Secure Messaging
  - X3DH Protocol
  - Symmetric-key Ratchet
  - Forward/Backward Secrecy
  - Diffie-Hellman Ratchet

“Case Studies” in the Course

- So far, we know digital signature (certificate), TLS (and PQTLS, KEM-TLS)…
- Real-world cryptographic applications are far more complex

- We will study several real-world cryptographic applications in this course

- Your final project (to be decided) **may use** some techniques from these real-world applications…

Secure Messaging

- Text Messages/Instant Messaging

WhatsApp

Signal

iMessage

End-to-End Encryption

- End-to-End Encryption (E2EE)
  - Only sender and recipient can decrypt messages…
  - **The server cannot decrypt messages** (if it does not tamper with the conversation…
  - Confidentiality and Privacy
  - In practice, the server will help relaying/forwarding messages…

End-to-End Encryption

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E2EE (by default)
Examples


WhatsApp


Signal


iMessage


Element

Non-E2EE (by default)
Examples


Discord


Facebook Messager


Telegram

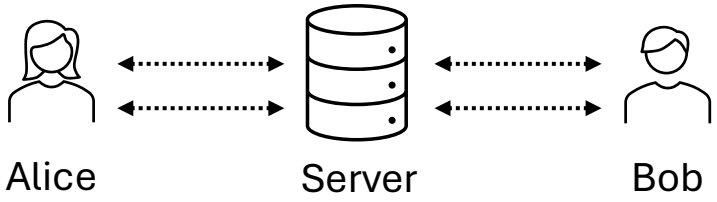
End-to-End Encryption

Initialization

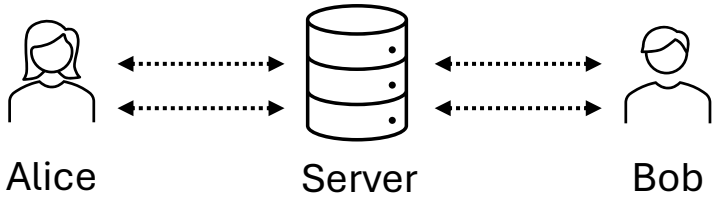
Including logging in, sharing users' information, ...

Messaging

E2EE



Non-E2EE

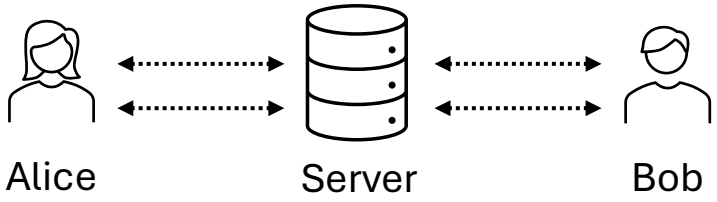


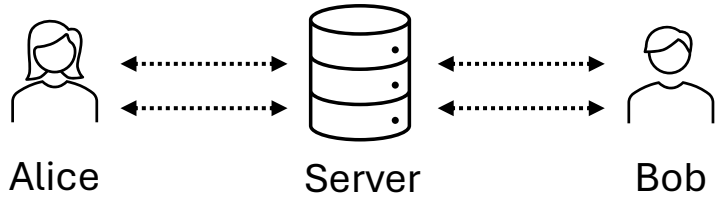
End-to-End Encryption

E2EE

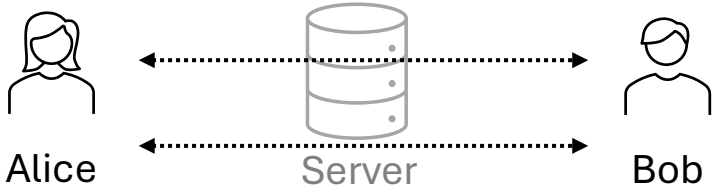
Non-E2EE

Initialization



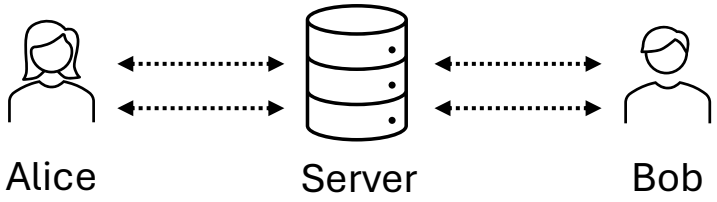
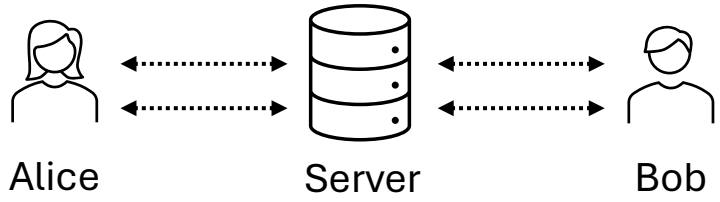
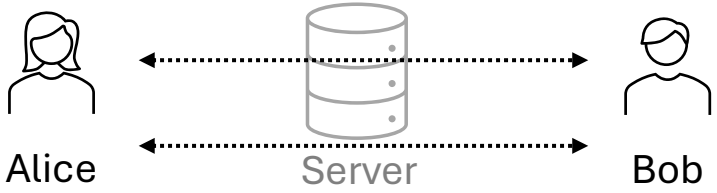
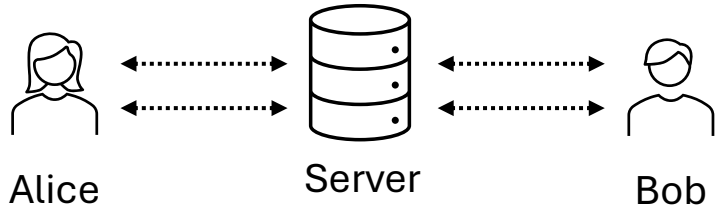


Messaging



The server **only relays** the encrypted messages, no storage (or just short-term storage).

End-to-End Encryption

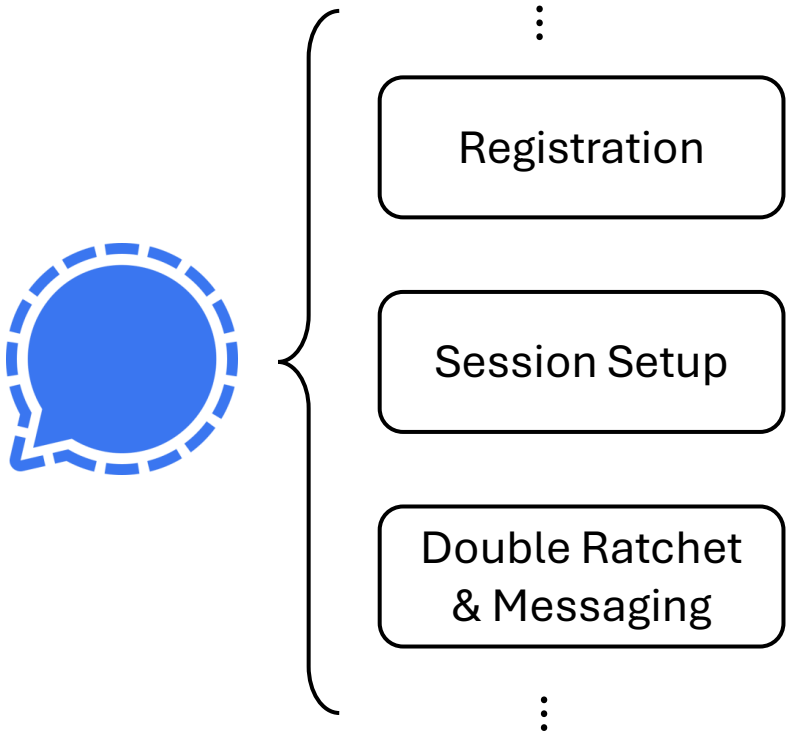
	E2EE	Non-E2EE
Initialization		
Messaging		
	The server **only relays** the encrypted messages, no storage (or just short-term storage).	Encrypted communication (e.g., via TLS) with the server, **but the messages may be stored (in plaintext)...**

Signal Secure Messaging Protocol

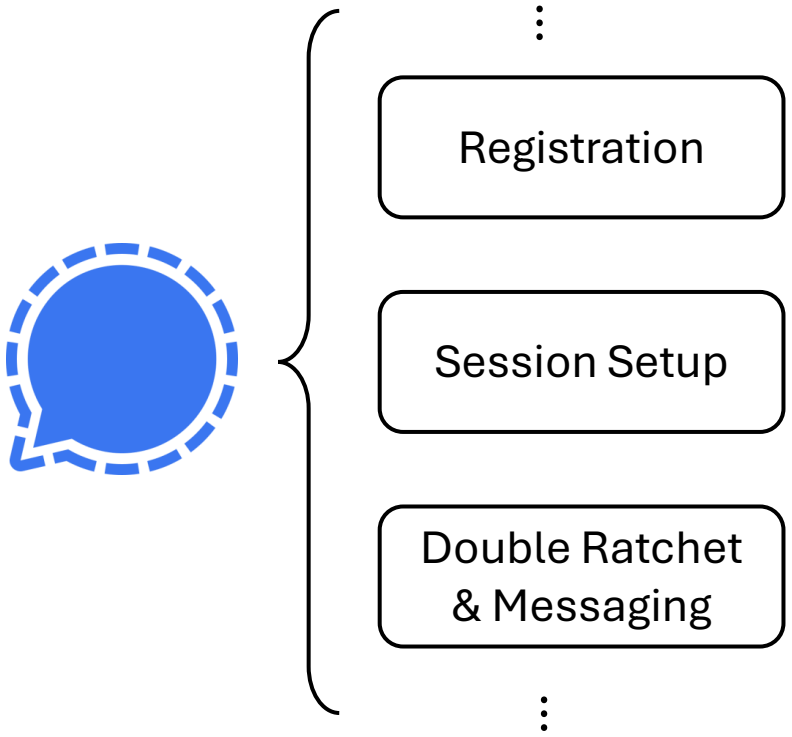
Signal

- One of the most secure instant messaging app
- End-to-end encryption (E2EE)
- WhatsApp also uses the Signal protocol

Signal Secure Messaging Protocol



Signal Secure Messaging Protocol



- Identity keys, signed pre-keys, one-time pre-keys, …
- X3DH (Extended Triple Diffie-Hellman) protocol **(Today)**
- Double Ratchet Algorithm:
  - Symmetric Ratchet **(Today)**
  - Diffie-Hellman Ratchet

The X3DH Protocol

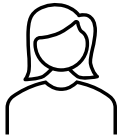
- Address *How to Establish Secure Initial Shared Secret*
  - It needs the server to help sharing pre-information
- Based on (EC)DH
- Mutual Authentication:
  - Two communication parties have long-term key pairs
- Forward Secrecy

The X3DH Protocol – Key Pairs

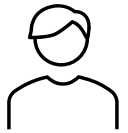
- Key pairs of each party:
  - For simplicity, we define 'XPK' always equals to 'g^{xk}'
  - All public keys (along with the user identity) will be stored in the server

Public parameters: $(\mathbb{G}, g, q)$:

A q-order EC group $\mathbb{G}$ with a generator g



Alice



Bob

	Alice	Bob
Identity secret key (IK)	$ik_{\mathrm{A}} \in_{\$} \mathbb{Z}_q$	$ik_{\mathrm{B}} \in_{\$} \mathbb{Z}_q$
Identity public key (IPK)	$IPK_{\mathrm{A}} (= g^{ik_A})$	IPK_{B}
Signing secret pre-key (SK)	$sk_{\mathrm{A}} \in_{\$} \mathbb{Z}_q$	$sk_{\mathrm{B}} \in_{\$} \mathbb{Z}_q$
Signing public pre-key (SPK)	SPK_{A}	SPK_{B}
One-time secret pre-keys (OK)	$\{ok_{\mathrm{A}}^1, ok_A^2, \ldots\} \subseteq_{\$} \mathbb{Z}_q$	$\{ok_{\mathrm{B}}^1, ok_B^2, \ldots\} \subseteq_{\$} \mathbb{Z}_q$
One-time public pre-keys (OPK)	$(OPK_{\mathrm{A}}^1, OPK_{\mathrm{A}}^2, \ldots)$	$(OPK_{\mathrm{B}}^1, OPK_{\mathrm{B}}^2, \ldots)$

The X3DH Protocol – Key Pairs

- Key pairs of each party:
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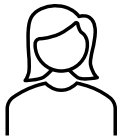
Identity keys

- Generated during registration
- Will be used for **Key Exchange and Signing**

Public parameters: $(\mathbb{G}, g, q)$:

A q-order EC group $\mathbb{G}$ with a generator g



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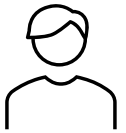

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The X3DH Protocol – Key Pairs

- Key pairs of each party:
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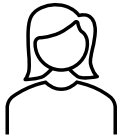
  - All public keys (along with the user identity) will be stored in the server

Signing Pre-keys

- Generated during registration
- Updated periodically (e.g., once a week, or once a month)
- Will be used for **Key Exchange and Signing**

Public parameters: $(\mathbb{G}, g, q)$:
A q-order EC group $\mathbb{G}$ with a generator g



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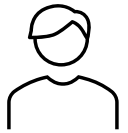

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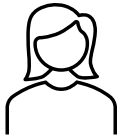

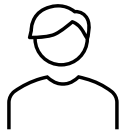

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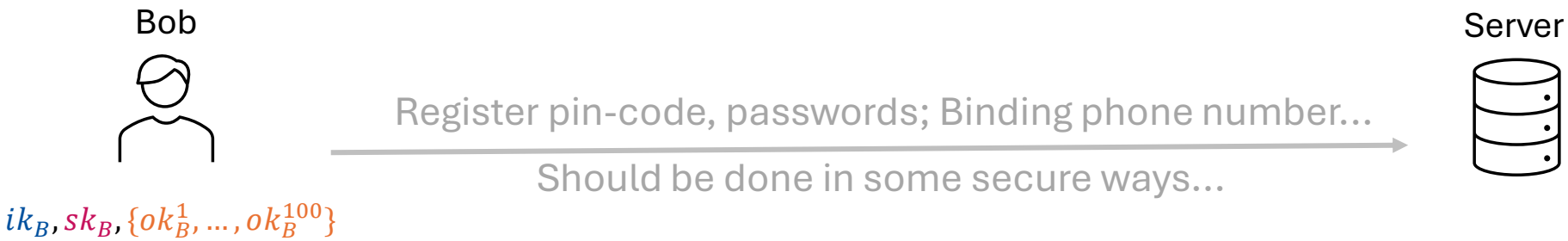
One-time Pre-keys

- Generated as a batch during registration
- Each key is used once for each new session; Deleted after use
- Re-generated when used up (or the supply is low)

The X3DH Protocol – Pre-key Bundles

- When Bob registers (we only focus on the cryptographic parts)…
  - For simplicity, we define 'XPK' always equals to 'g^{xk}'



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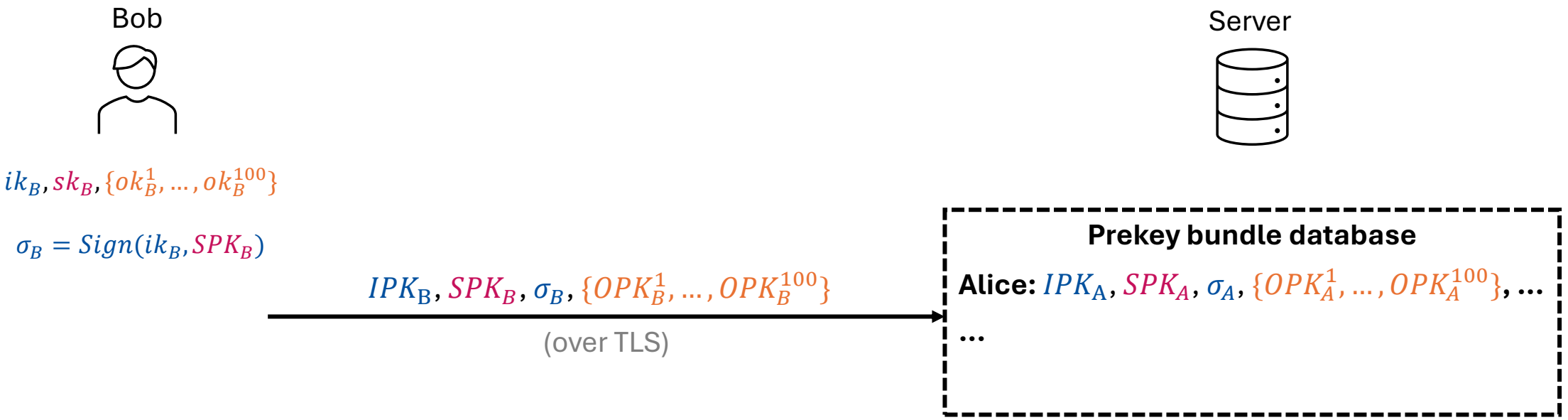

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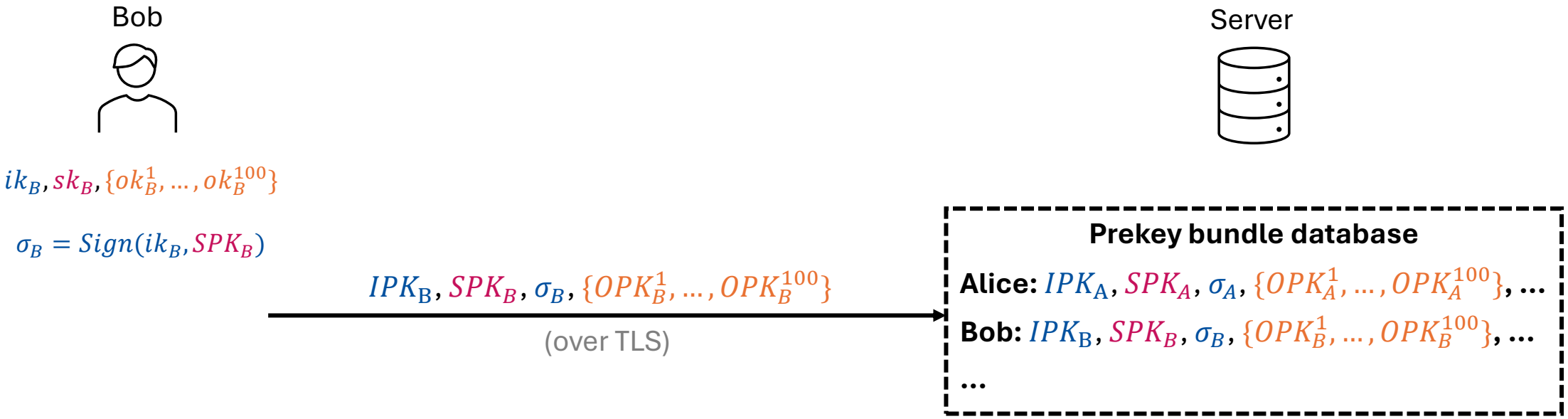

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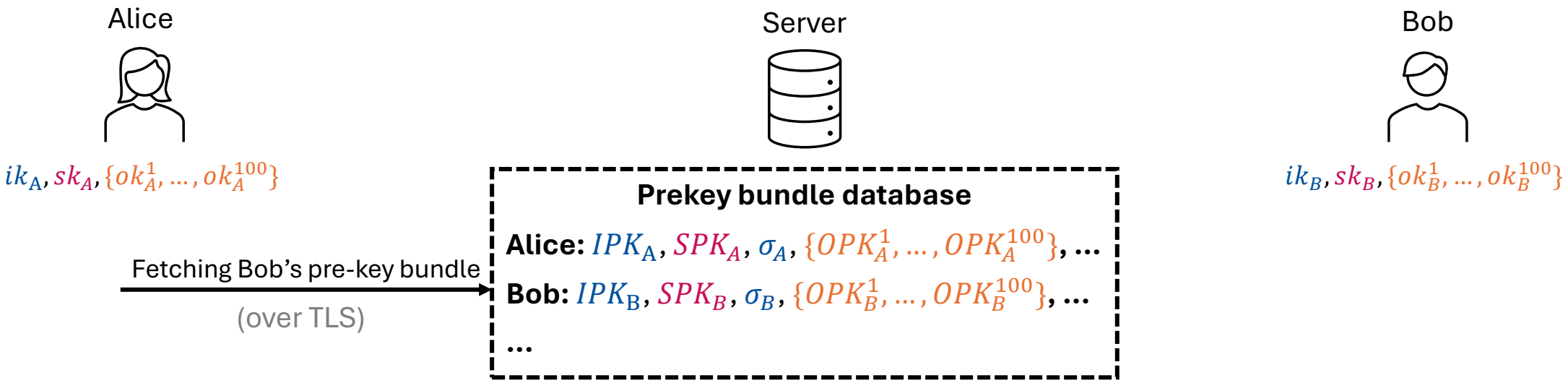
- When Alice communicates with Bob…



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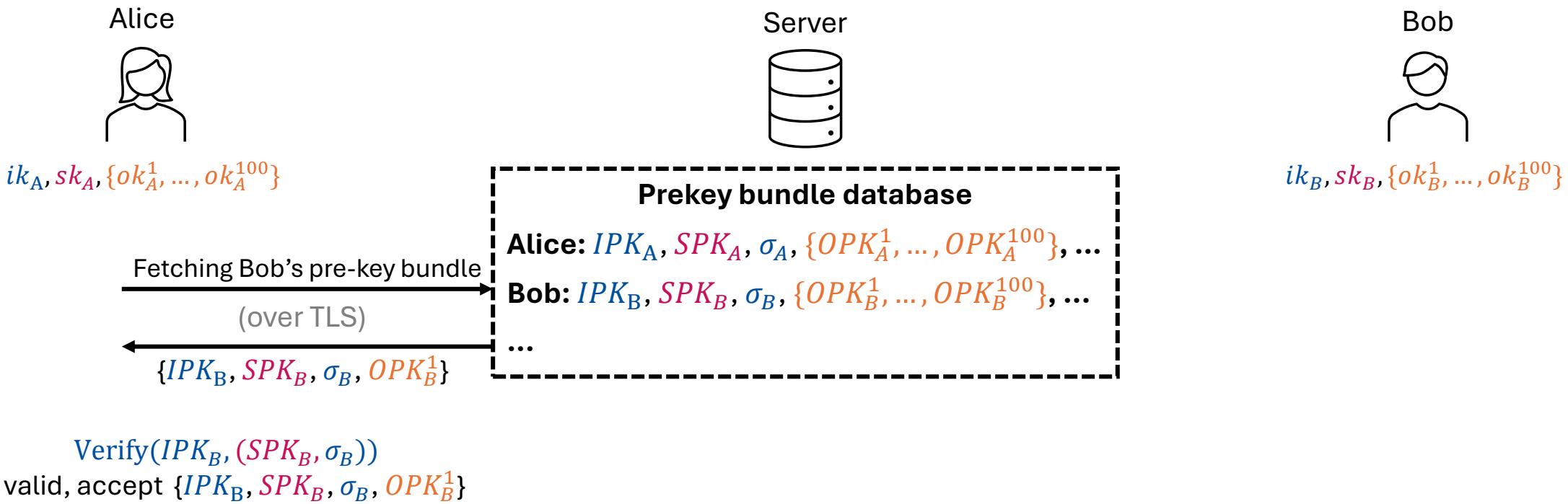


$\text{Verify}(IPK_B, (SPK_B, \sigma_B))$
if valid, accept $\{IPK_B, SPK_B, \sigma_B, OPK_B^1\}$

The X3DH Protocol

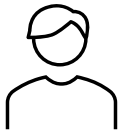
- When Alice communicates with Bob…

Alice

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$\{IPK_B, SPK_B, \sigma_B, OPK_B^1\}$

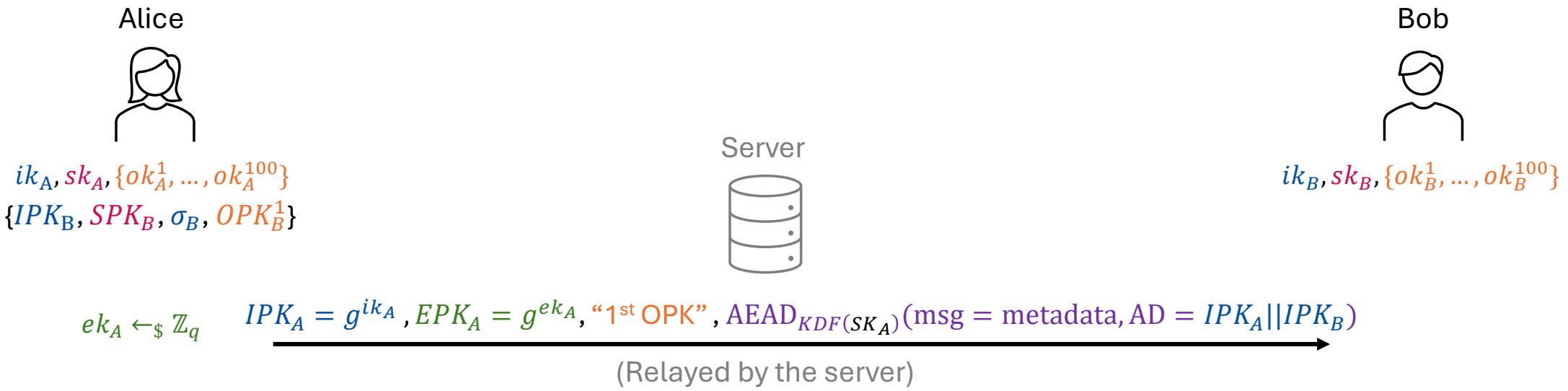
Bob



$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

The X3DH Protocol

- When Alice communicates with Bob…



$SK_A = \text{X3DH\_Key\_Alice}(ik_A, ek_A, IPK_B, SPK_B, OPK_B^1)$
(…will be explained later)

The X3DH Protocol

- When Bob receives messages (which is actually relayed by the server) from Alice…

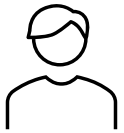
Alice

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$\{IPK_B, SPK_B, \sigma_B, OPK_B^1\}$

Server

Bob



$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

$ek_A \leftarrow_\$ \mathbb{Z}_q$

$IPK_A = g^{ik_A}, EPK_A = g^{ek_A}$, "1st OPK", $\text{AEAD}_{KDF(SK_A)}(\text{msg} = \text{metadata}, \text{AD} = IPK_A || IPK_B)$

(Relayed by the server)

Accept if the initial ciphertext can be decrypted

$SK_A = \text{X3DH\_Key\_Alice}(ik_A, ek_A, IPK_B, SPK_B, OPK_B^1)$
(…will be explained later)

$SK_B = \text{X3DH\_Key\_Bob}(IPK_A, EPK_A, ik_B, sk_B, ok_B^1)$
(…will be explained later)

The X3DH Protocol

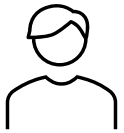
- How the X3DH protocol computes a shared secret…

Alice

X3DH_Key_Alice(ik_A, ek_A, IPK_B, SPK_B, OPK_B)

1. $DH_1 = SPK_B^{ik_A}$
2. $DH_2 = IPK_B^{ek_A}$
3. $DH_3 = SPK_B^{ek_A}$
4. $DH_4 = (OPK_B)^{ek_A}$
5. return $SK_A = KDF(DH_1, DH_2, DH_3, DH_4)$

Bob



SK_B = X3DH_Key_Bob(IPK_A, EPK_A, ik_B, sk_B, ok_B)

1. $DH_1 = IPK_A^{sk_B}$
2. $DH_2 = EPK_A^{ik_B}$
3. $DH_3 = EPK_A^{sk_B}$
4. $DH_4 = EPK_A^{ok_B}$
5. return $SK_B = KDF(DH_1, DH_2, DH_3, DH_4)$

The X3DH Protocol

- How the X3DH protocol computes a shared secret…

Alice

X3DH_Key_Alice(ik_A, ek_A, IPK_B, SPK_B, OPK_B)

1. $DH_1 = SPK_B^{ik_A}$
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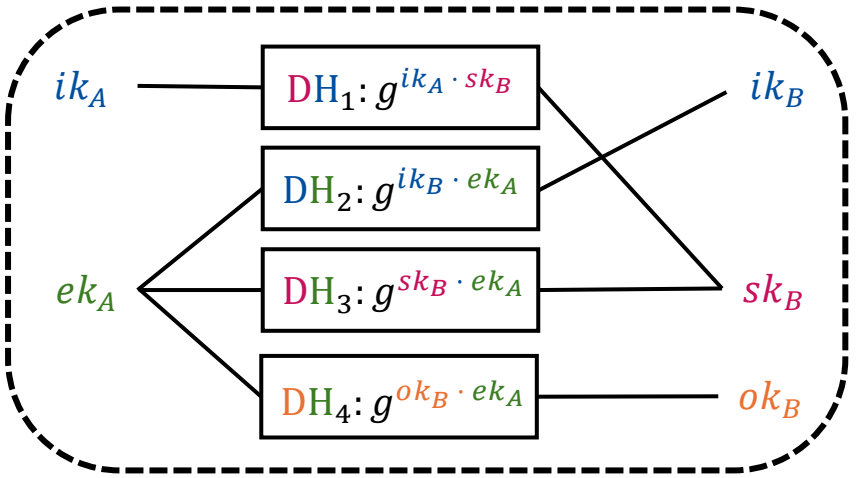
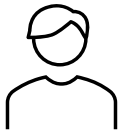


Bob



X3DH_Key_Bob(IPK_A, EPK_A, ik_B, sk_B, ok_B)

1. $DH_1 = IPK_A^{sk_B}$
2. $DH_2 = EPK_A^{ik_B}$
3. $DH_3 = EPK_A^{sk_B}$
4. $DH_4 = EPK_A^{ok_B}$
5. $SK_B = KDF(DH_1, DH_2, DH_3, DH_4)$

The X3DH Protocol

- Based on (EC)DH
- Trusted server required
  - Store public keys, relay messages, …
  - But it cannot decrypt ciphertexts…
- 0-RTT (Zero round-trip time)
  - Immediate message sending without waiting for a response
- Support offline communication
  - Can be executed even if Bob (the receiver) is offline
  - Offline messages (encrypted) will be stored in the server until Bob is online again
- Mutual Authentication, Forward Secrecy, …
  - In this course, we focus on *How it works* rather than *Why it is secure*…

X3DH

The X3DH Protocol

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X3DH

A note: Do not confuse X3DH with TLS

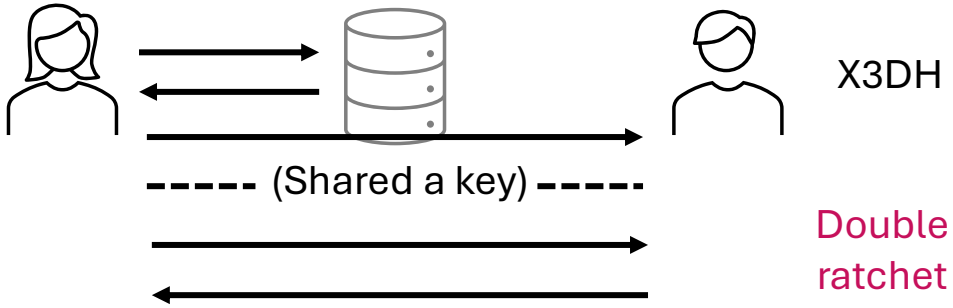
Different primary goals/settings:

X3DH: secure messaging between users, rely on trusted pre-shared public keys…

TLS: secure connections with a server, rely on trusted CAs and use certificates…

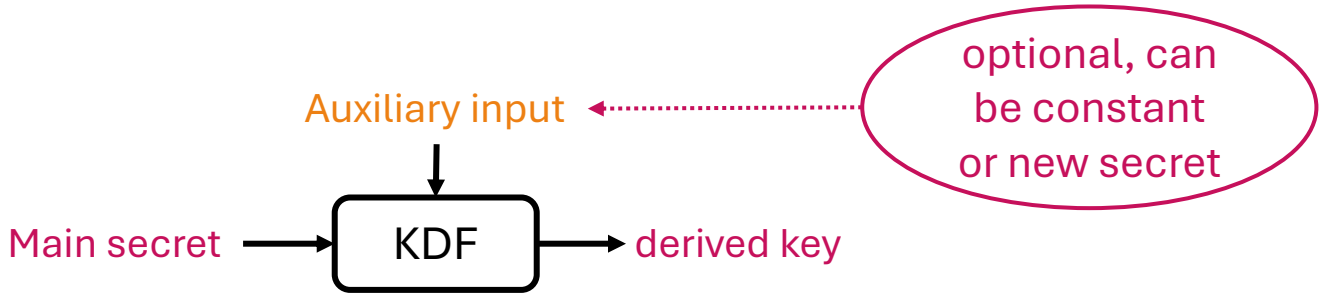
Double Ratchet

- After completing X3DH…
- … we use **Double Ratchet** to:
  - Encrypt messages + updates the shared key
  - ~~Encrypt messages using the same shared key~~
  - **Diffie-Hellman Ratchet** + **Symmetric-key Ratchet**
- Essential for forward/backward secrecy



Symmetric-key Ratchet

- KDF chain
  - KDF: Key derivation function



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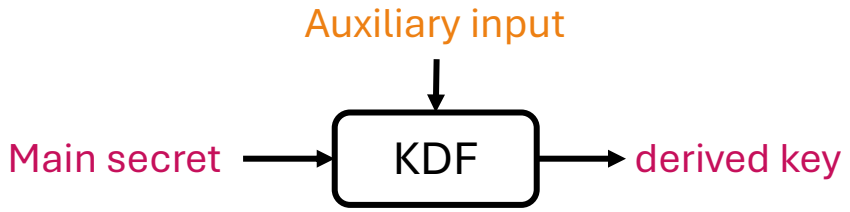
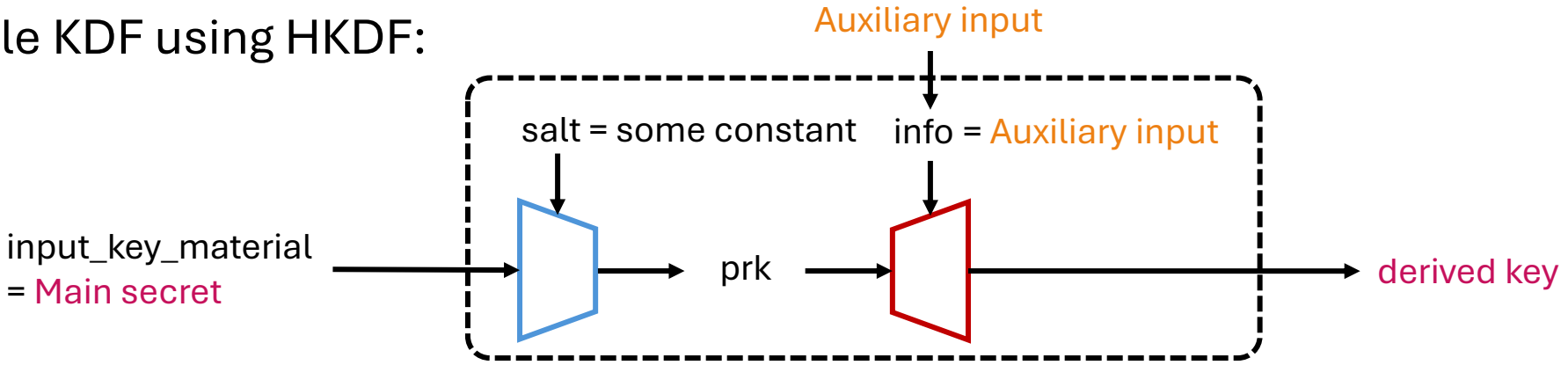


- Example KDF using HKDF:



1. prk = HKDF.**Extract**(input_key_material = Main secret, salt = some constant)
2. derived key = HKDF.**Expand**(prk, Auxiliary input)

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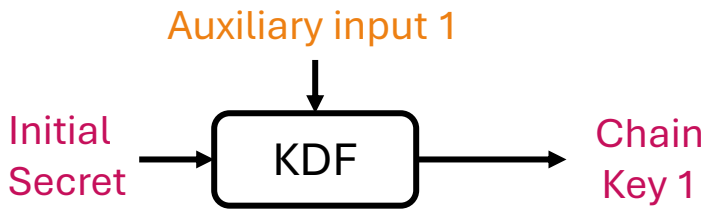

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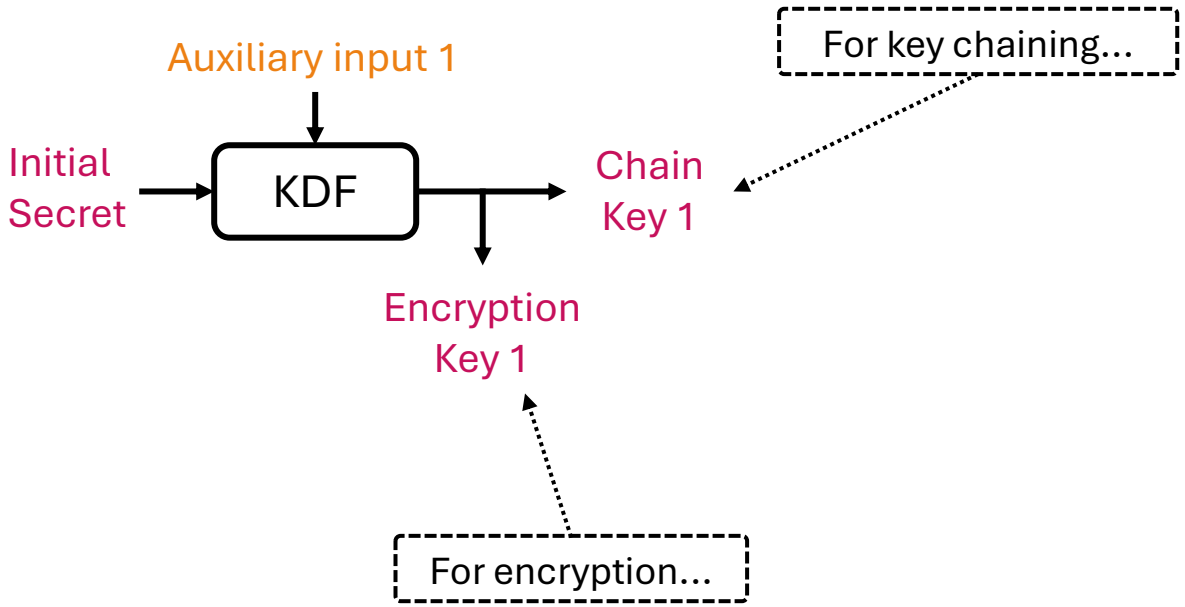

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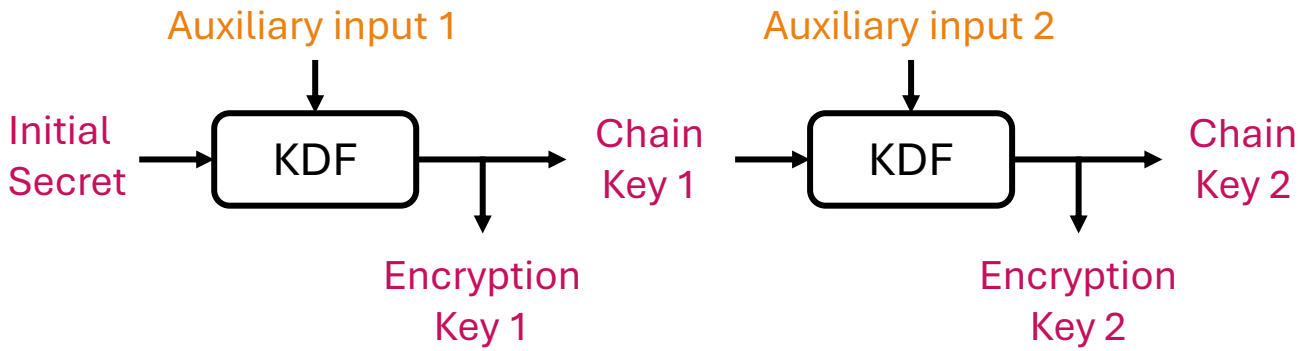

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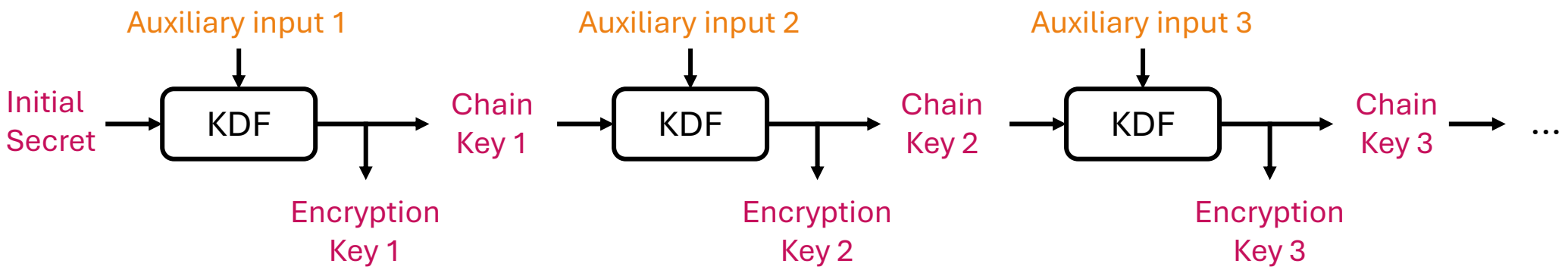

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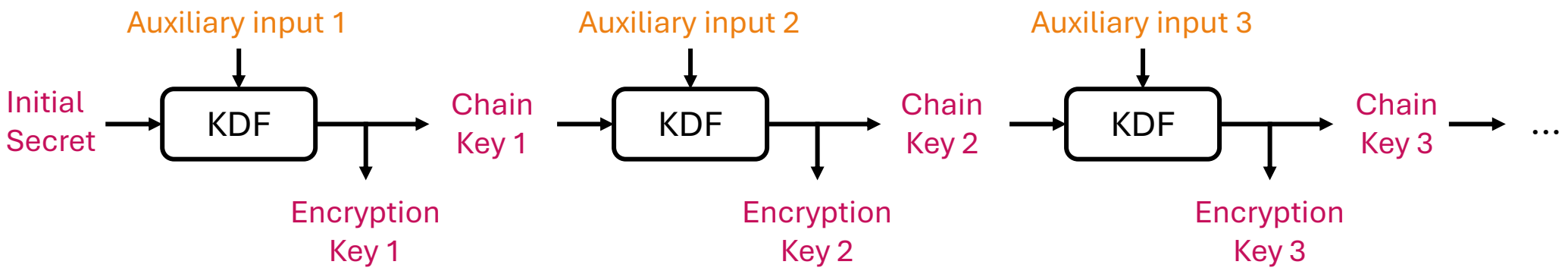

- Use Key Chain to encrypt messages

Symmetric-key Ratchet

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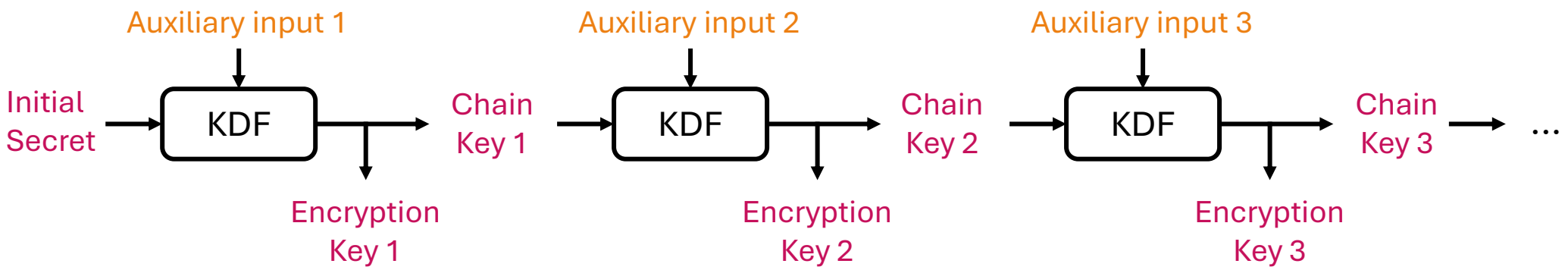
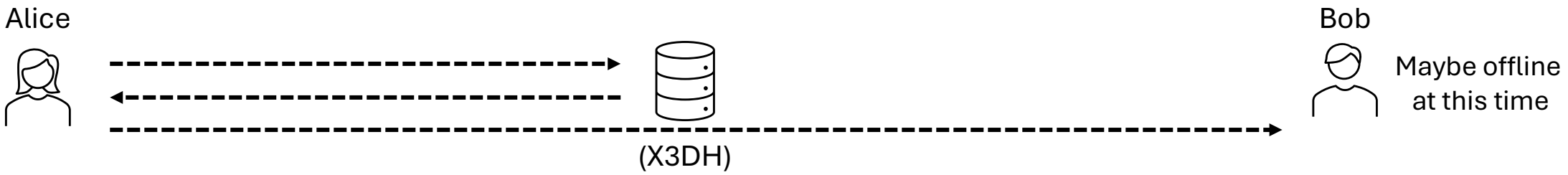


Symmetric-key Ratchet

- A toy example of secure messaging using symmetric-key ratchet



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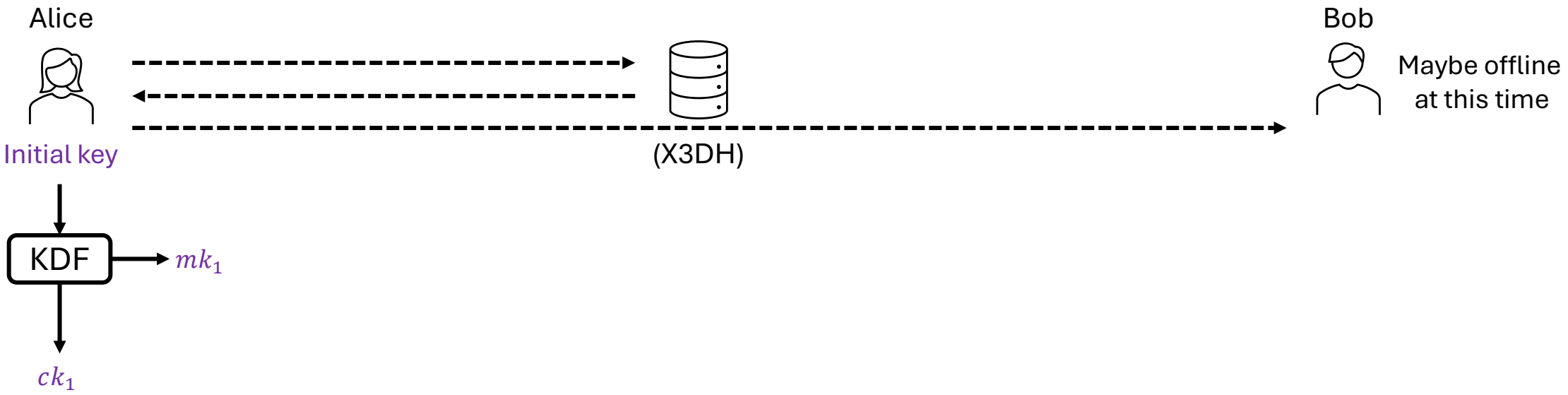

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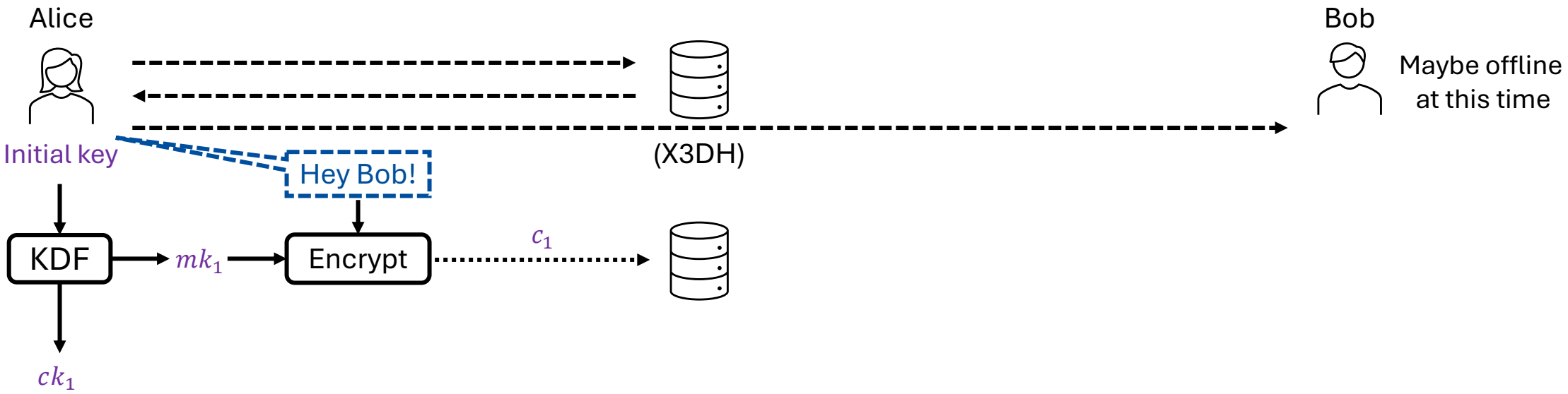

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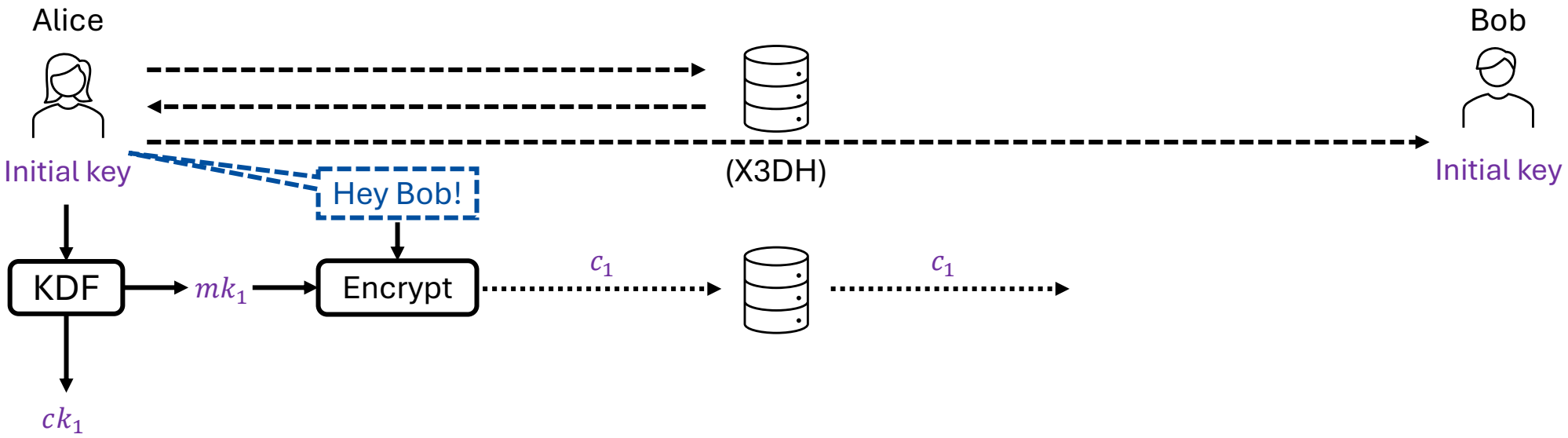

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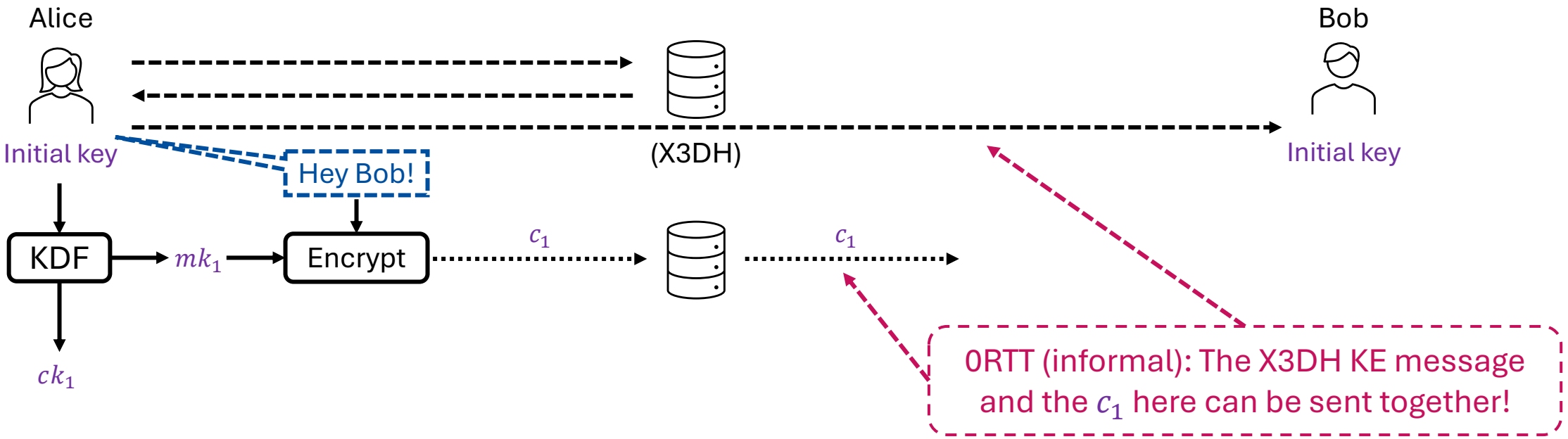

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Symmetric-key Ratchet

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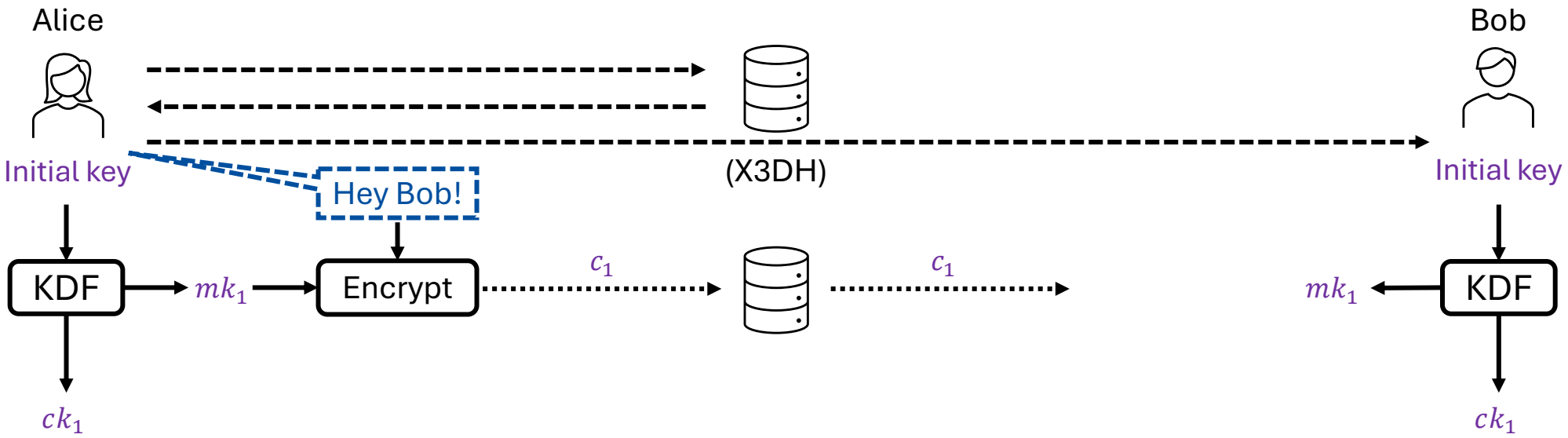

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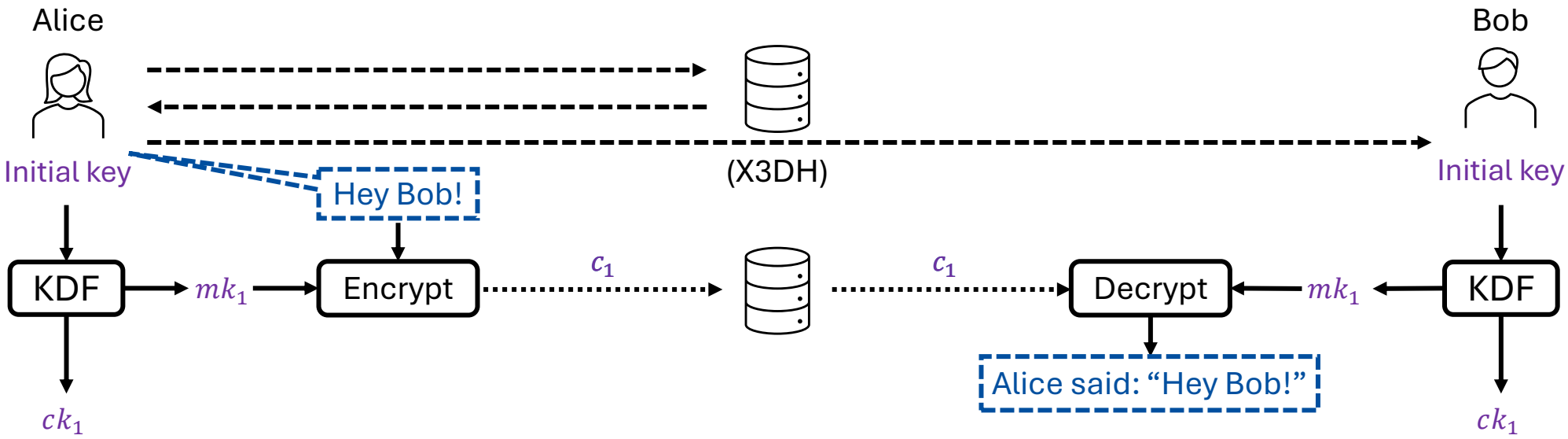

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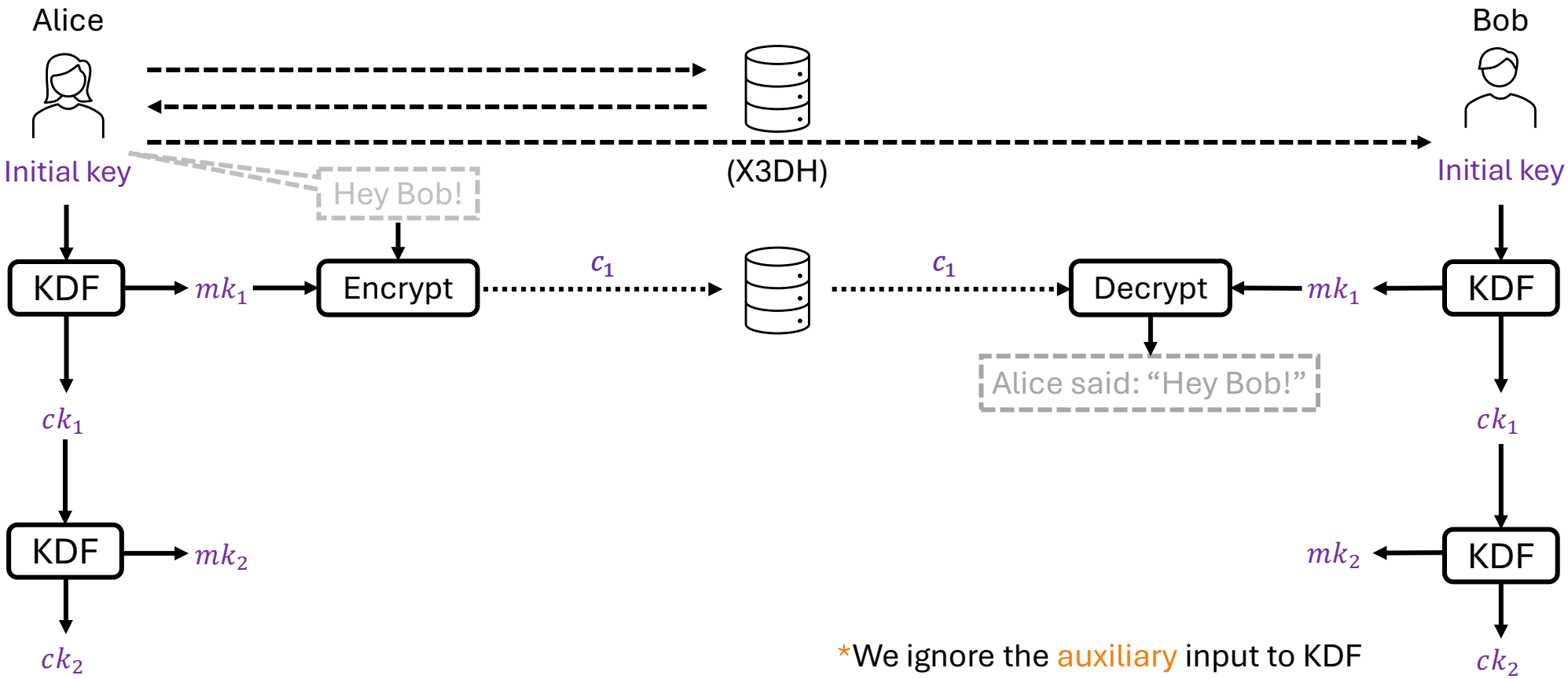

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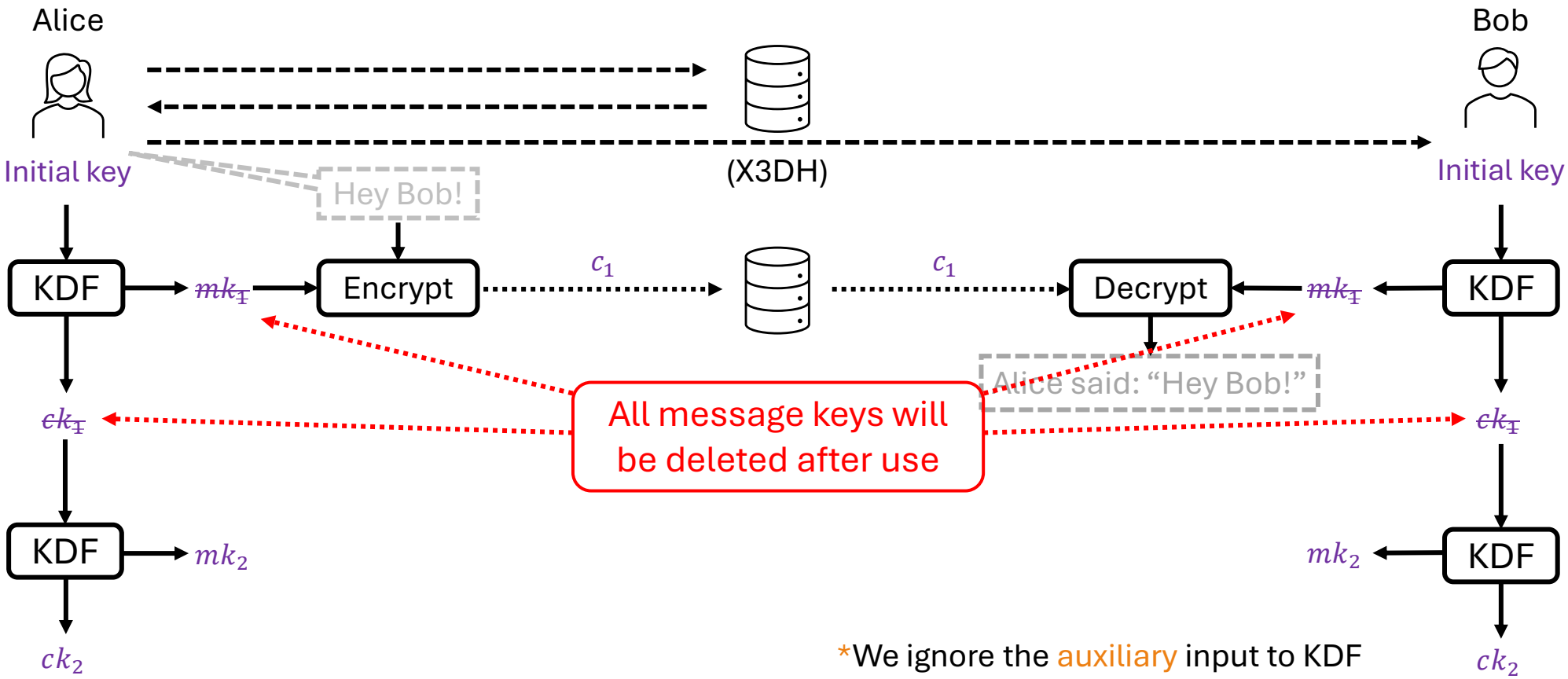

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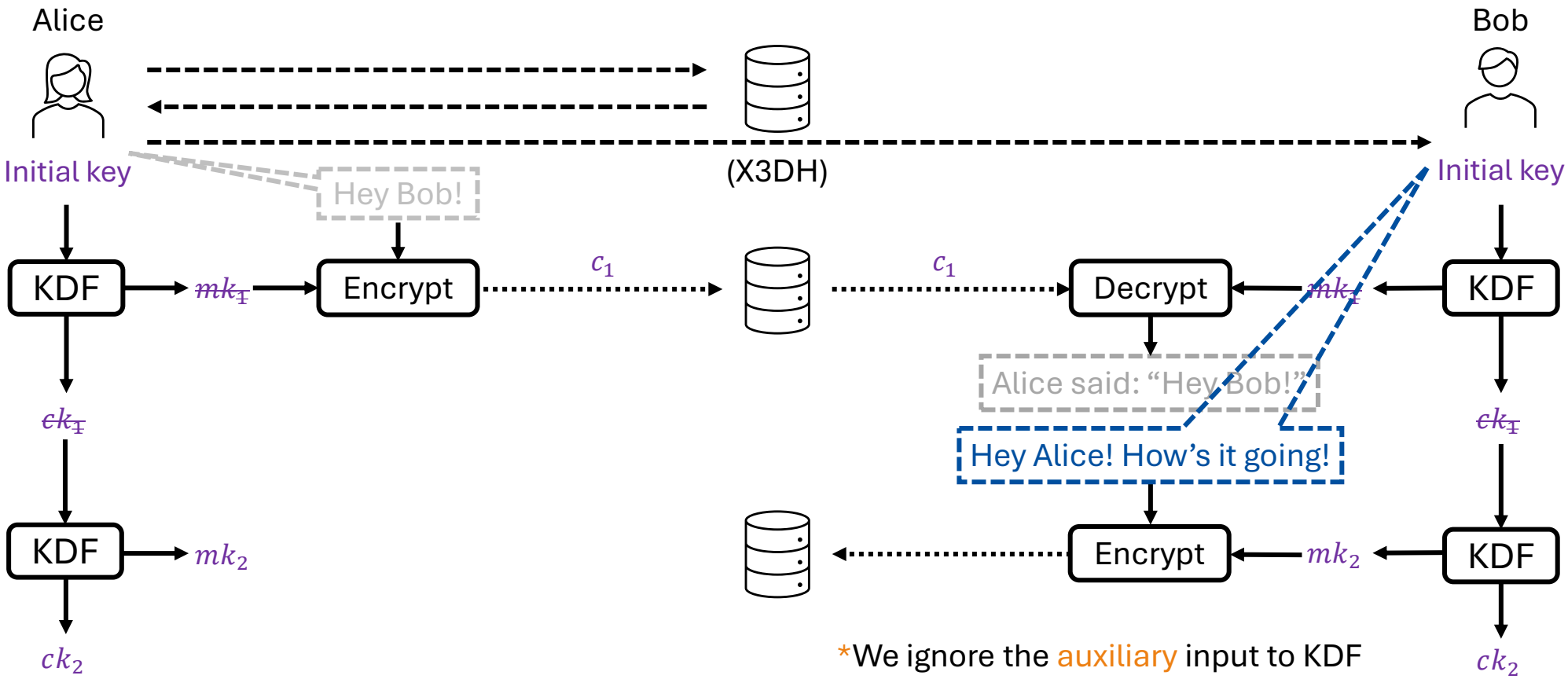

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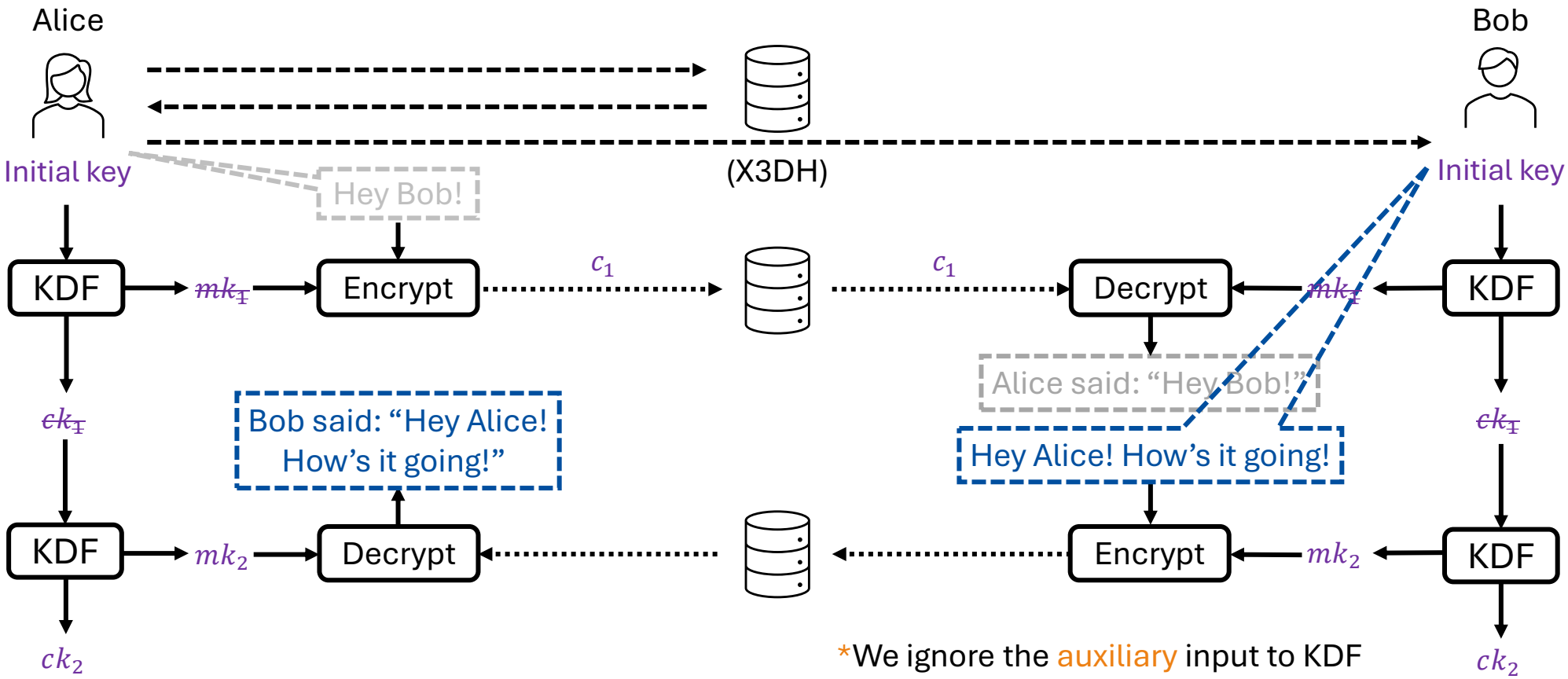

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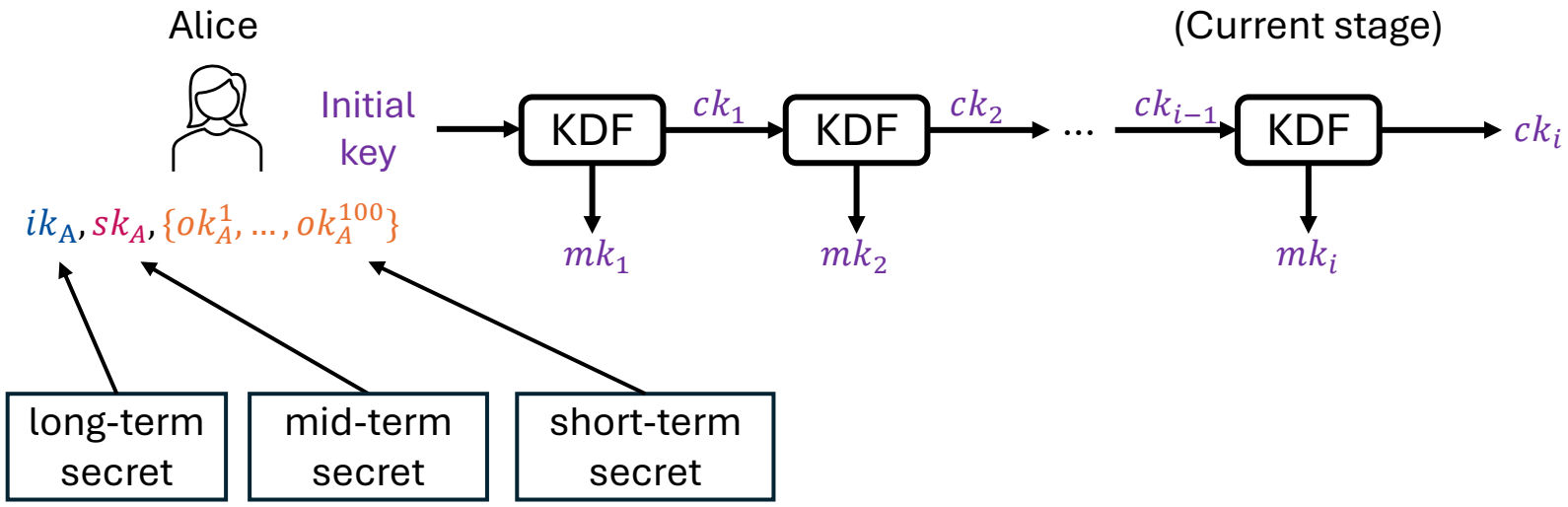
Forward Secrecy

- Long-term secret keys are compromised, but past communication remains secure…



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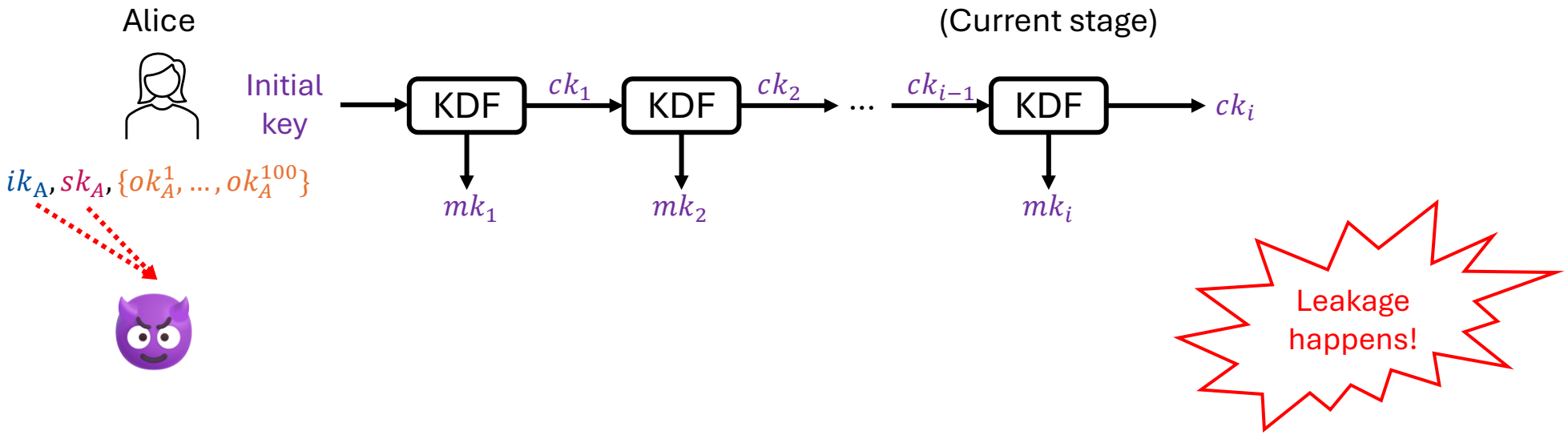

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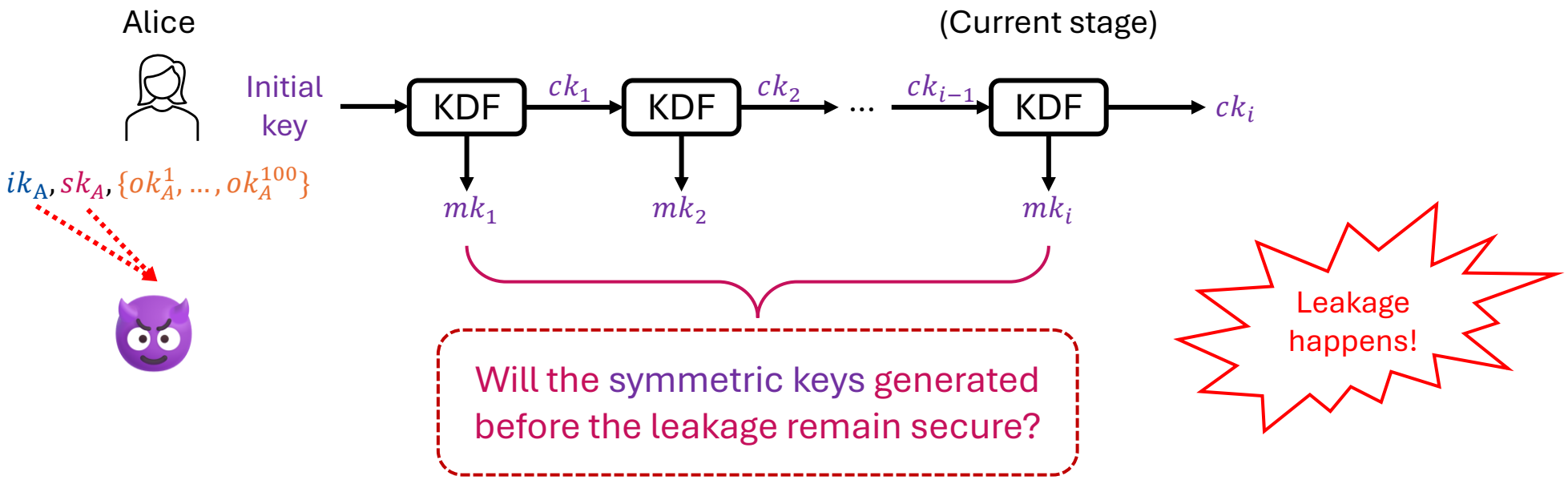
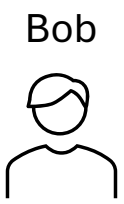


Forward Secrecy

- Recall: How the X3DH protocol computes a shared secret…



$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

$SK_A = \text{X3DH\_Key\_Alice}(ik_A, ek_A, IPK_B, SPK_B, OPK_B)$

1. $DH_1 = SPK_B^{ik_A}$
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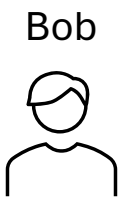

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ek_A is not a long-term secret

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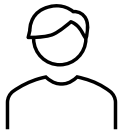
Alice

Leakage!

Bob



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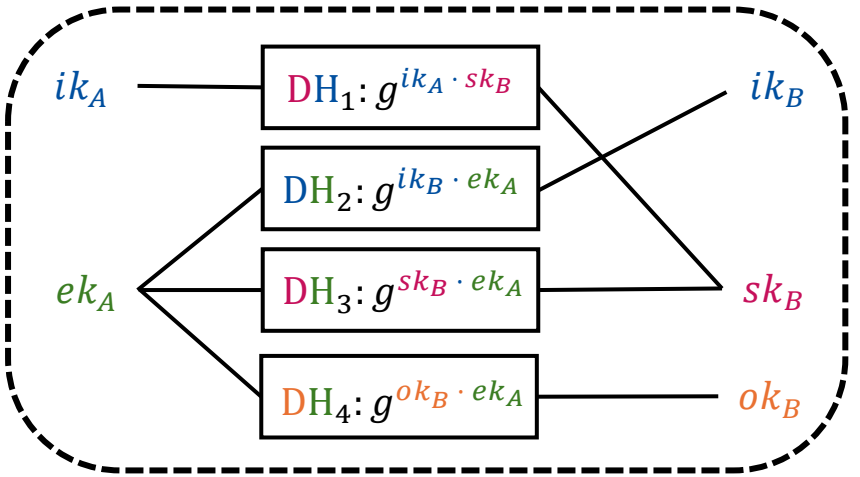

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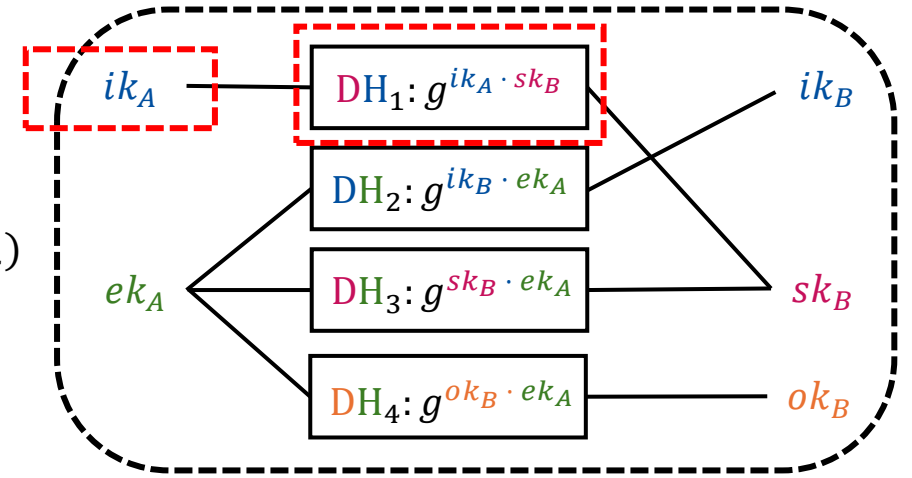
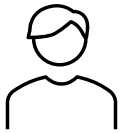


Bob



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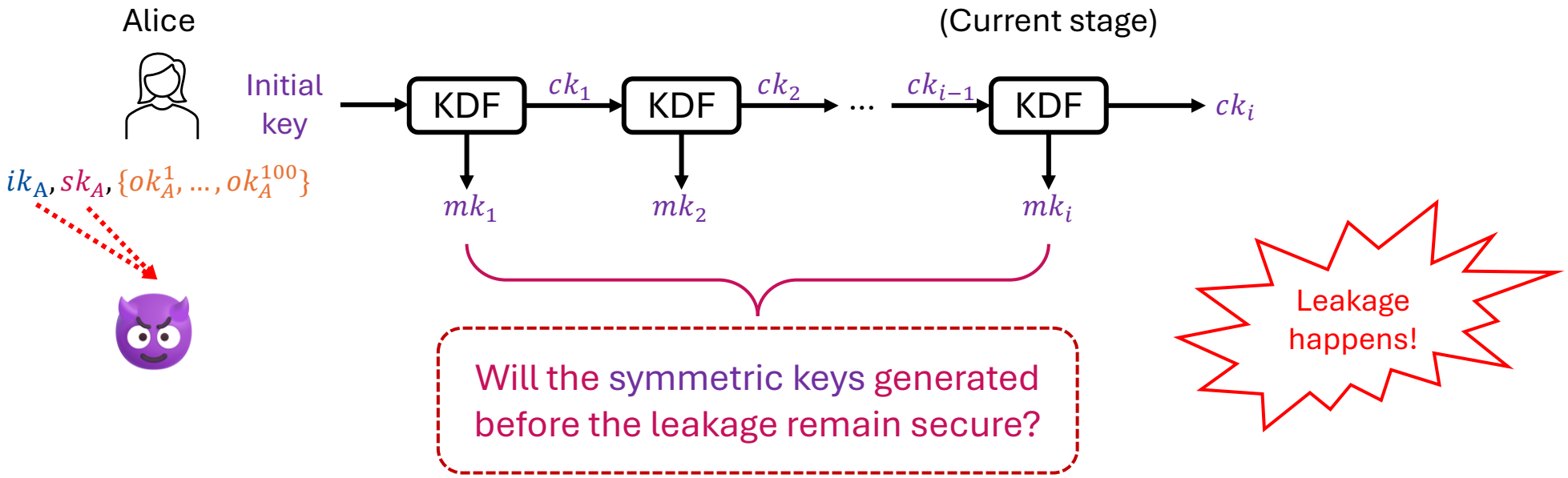

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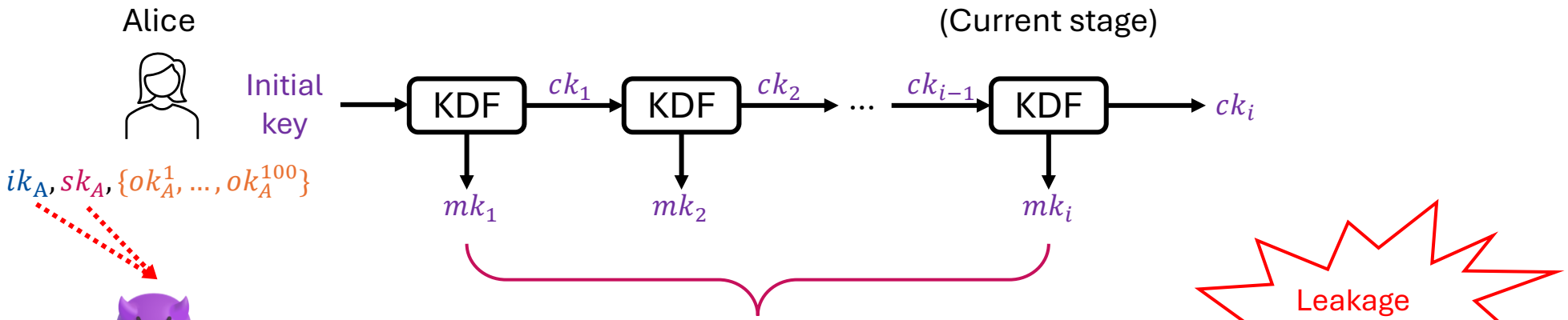

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DH_1 is not secure

But DH_2,3,4 remain secret

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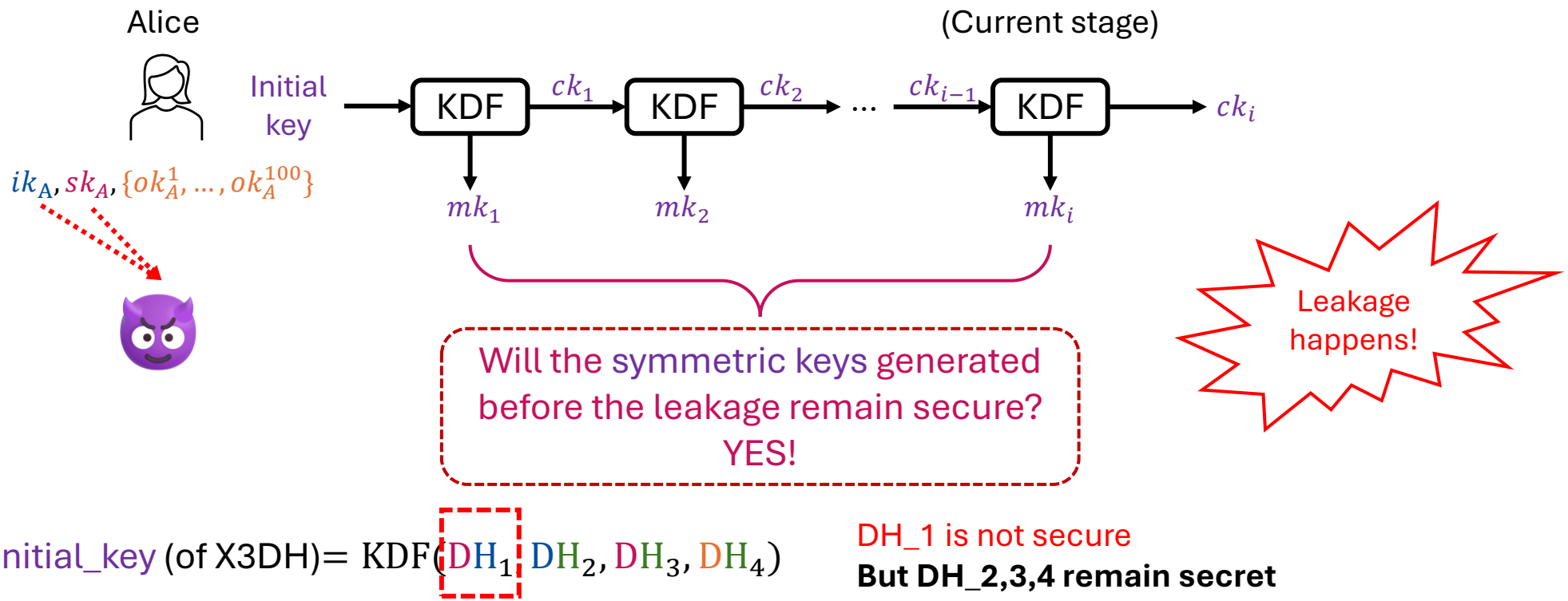

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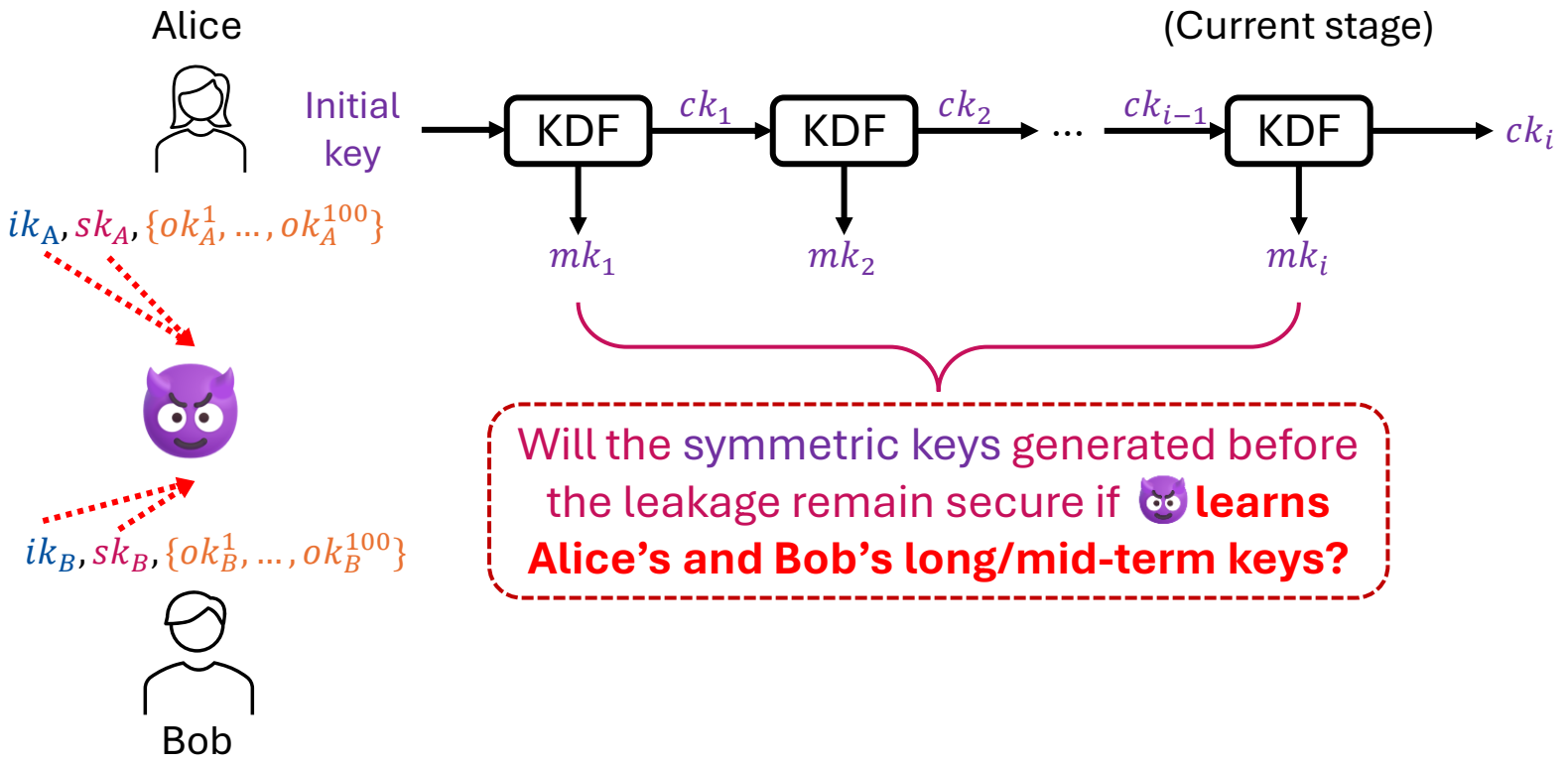

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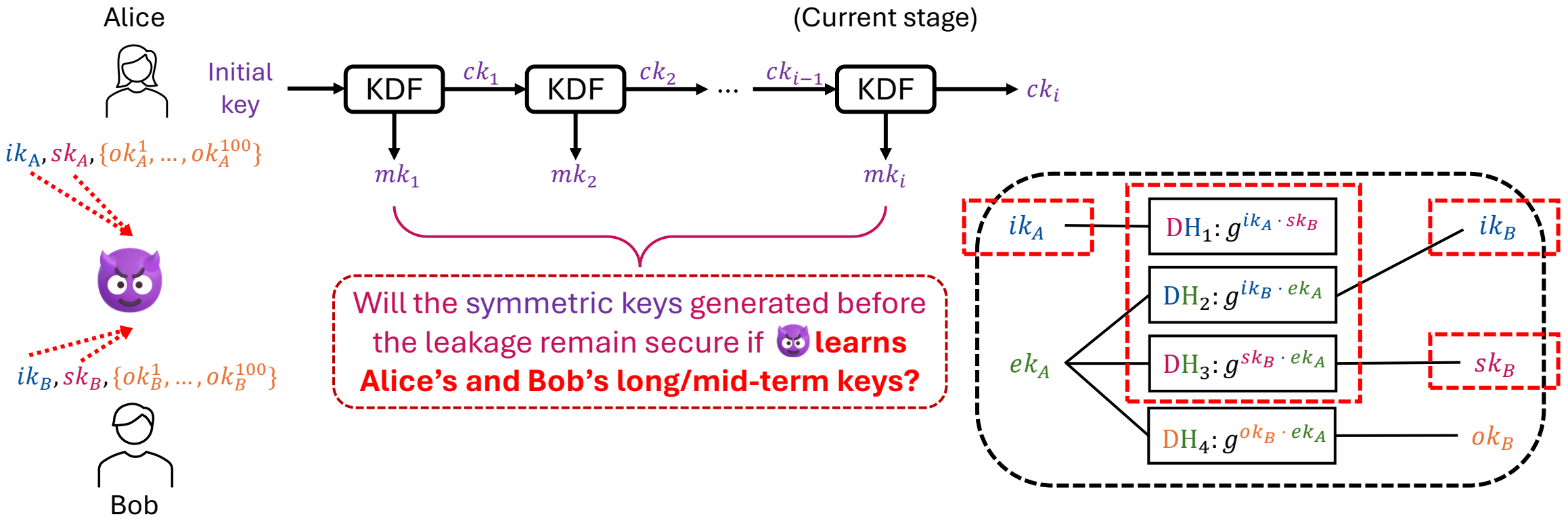

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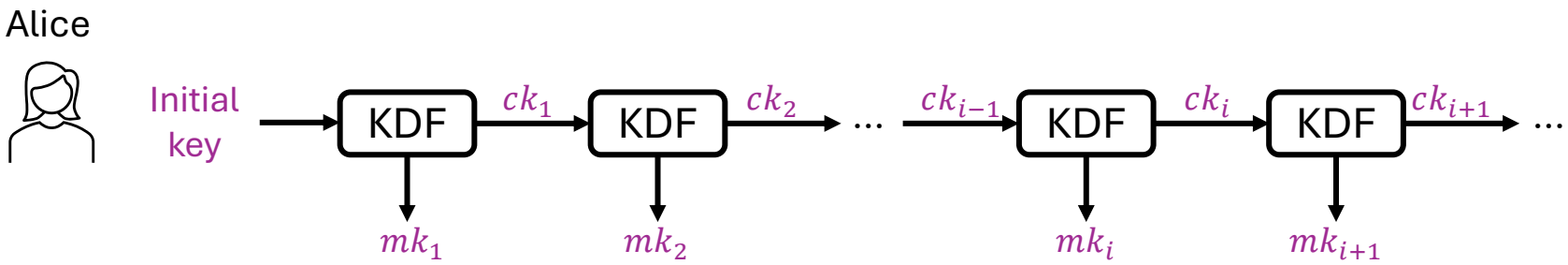

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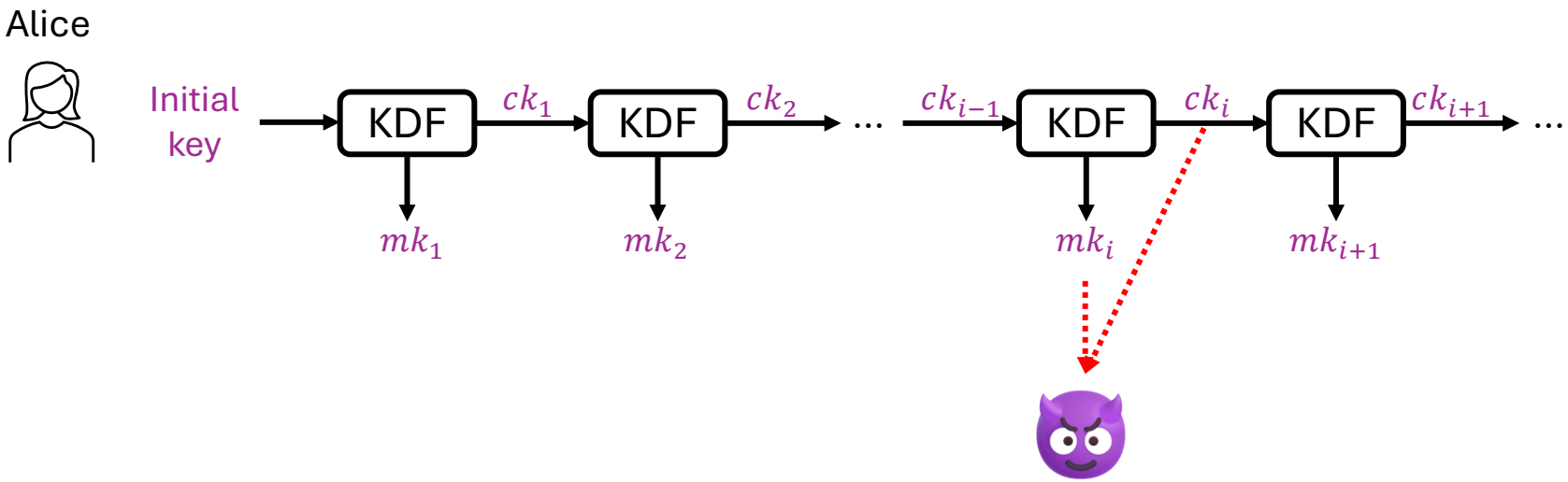

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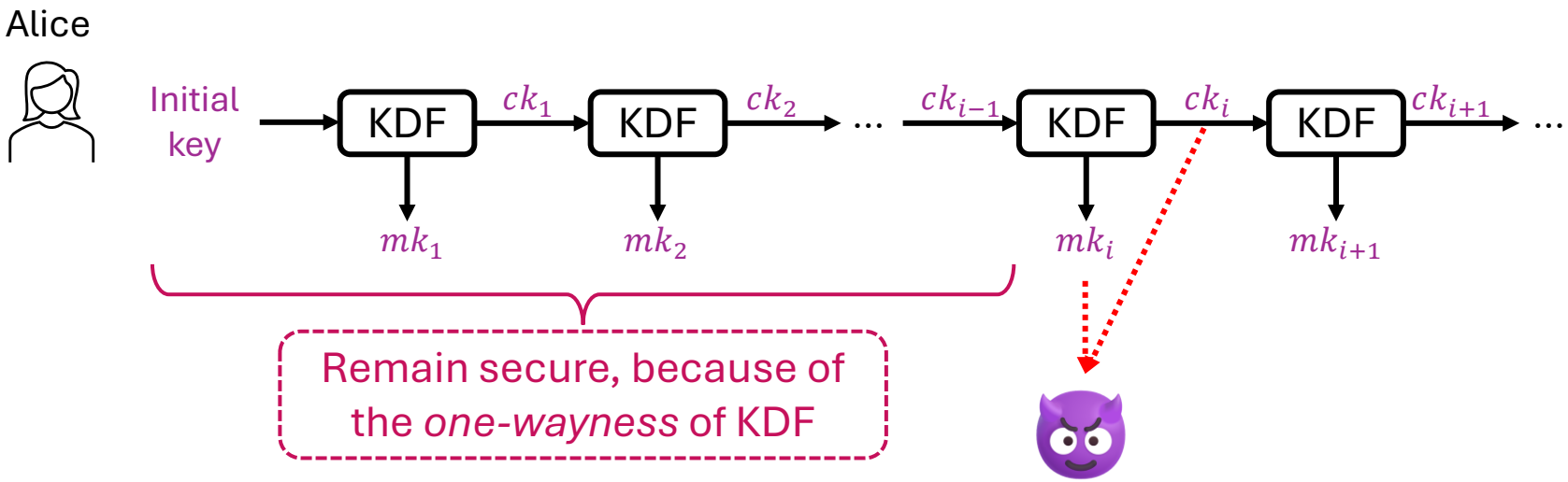

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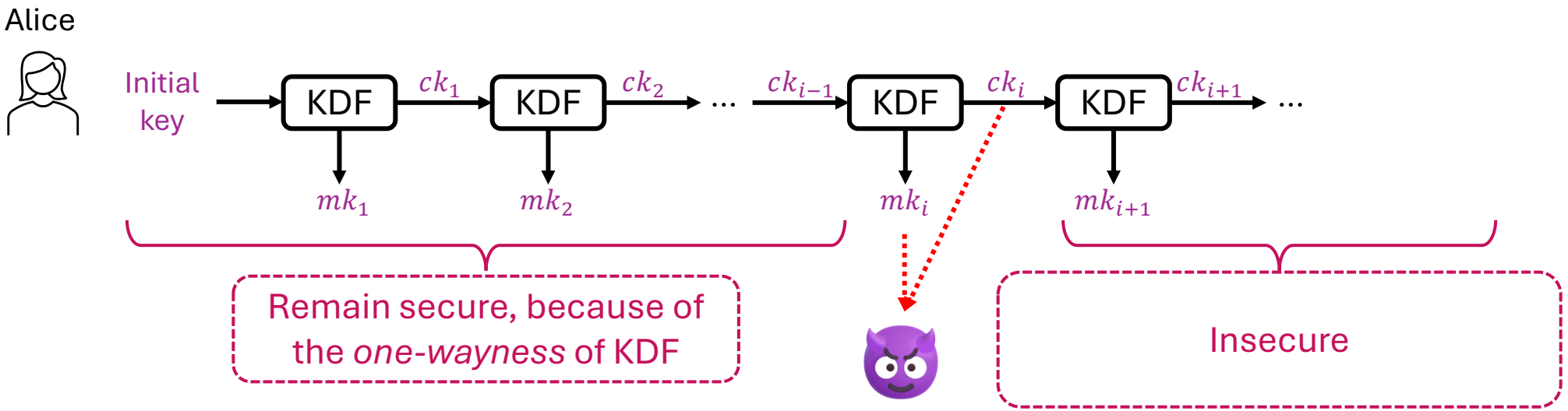
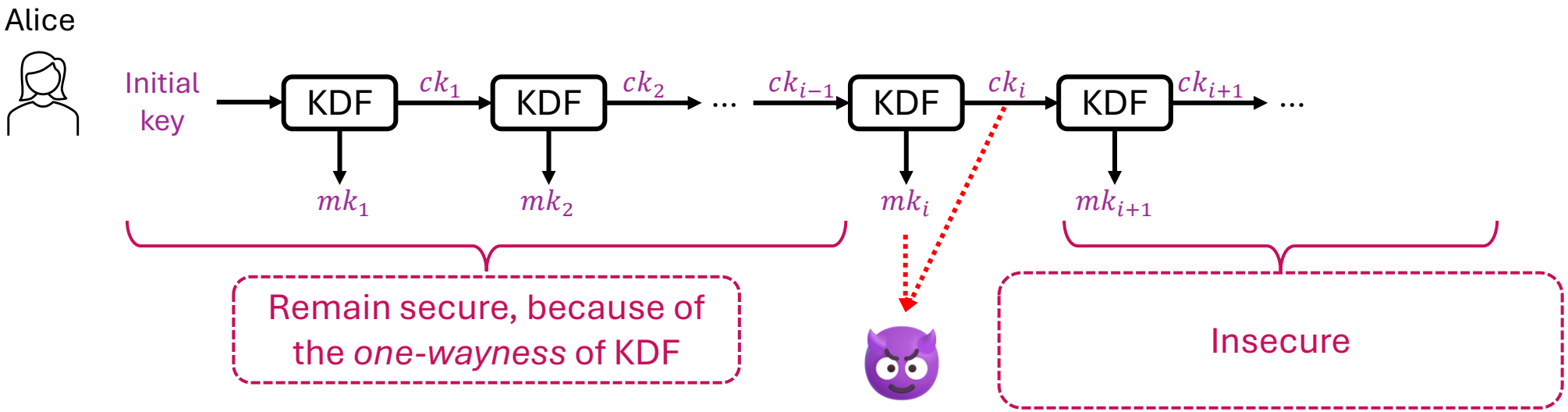


Backward Secrecy

- Future communication remains secure even if a current session key is compromised



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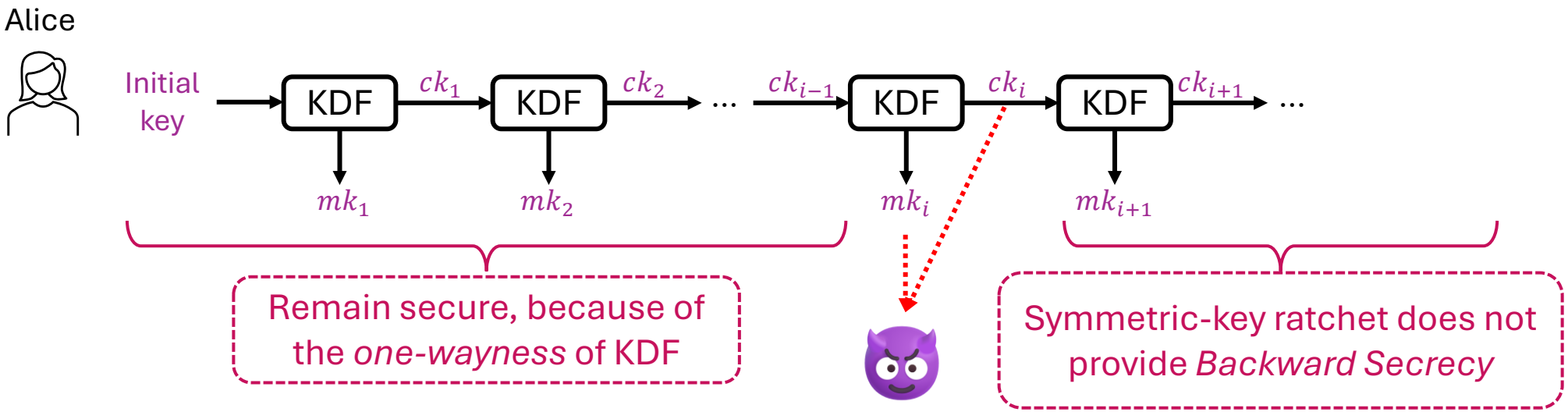

Diffie-Hellman Ratchet

- X3DH + Symmetric-key Ratchet
  - X3DH provides *Forward Secrecy*
  - Current session key compromises does not lead to the compromise of previous session keys
    - (by the one-wayness of KDF in Symmetric-key Ratchet)
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- Solution: Diffie-Hellman Ratchet (Next lecture)

Exercise

- Implement an X3DH demo
  - **Registration:** Alice and Bob register their key pairs with the server
  - **Fetch pre-key bundles:** To run the key exchange with Bob, Alice first fetches his pre-key bundle from the server
  - **Key Exchange:** After receiving Bob's pre-key bundle, Alice runs the X3DH key exchange to get the shared secret
- Implement the KDF chain (use the X3DH shared secret as the initial secret)
  - Example (using HMAC-KDF-chain): $ck_{i+1} = HMAC(ck_i, \text{"chain key"}), mk_i = HMAC(ck_i, \text{"message key"})$
- Implement the secure messaging demo using X3DH and KDF
  - Alice starts the conversation
  - If Bob is offline, Alice's protocol messages are stored in the server
  - Alice starts to derive the initial chain key and root key using $(rk, ck_0) = HKDF([X3DH\ secret], "X3DH")$, and send encrypted messages to Bob. (Root key is ignored now)
  - Once Bob is online, the server sends Alice's protocol messages to Bob, and Bob decrypts Alice's messages and responds…