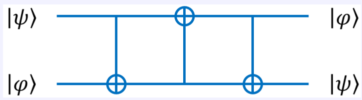
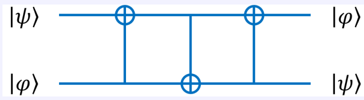
Exercises – Week 6

Exercise 1: Quantum Swap (qSWAP) gate

Let $|\psi\rangle$ and $|\varphi\rangle$ be two single-qubit state. Show that the following circuits can exchange the states of $|\psi\rangle$ and $|\varphi\rangle$ even if they are in superposition:



Furthermore, show that the following circuit also implements qSWAP:



Exercise 2: Decompose the state of a composite system

Let $|\psi_3\rangle := \frac{1}{\sqrt{6}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle)$ be a three-qubit state. Try to find a two-qubit state $|\psi_2\rangle$ and a single-qubit state $|\psi_1\rangle$ such that $|\psi_2\rangle \otimes |\psi_1\rangle = |\psi_3\rangle$.

Exercise 3: Projective measurement with the observable **I**

Let $\mathbf{I}$ be the identity operator of the Hilbert space $\mathbb{C}^{2^n}$. If we use $\mathbf{I}$ as the observable for an projective measurement, then what do the projectors in $\mathbf{I}$ look like? (Hint: For $n = 1$, $\mathbf{I} = |0\rangle\langle 0| + |1\rangle\langle 1|$) Moreover, explain the following sentence:

If we use the identity operator $\mathbf{I}$ *as an observable, then the projective measurement has no physical meaning.*

Exercise 4: Observable of measurement in the computational basis

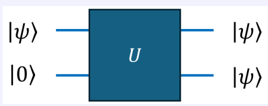
Find an observable for projective measurement in the computational basis of $\mathbb{C}^{2^n}$ (n-qubit system).

Exercise 5: Permutation on a uniform state

Let $f_{\mathbf{a}}$ be a function that $f(\mathbf{x}) := \mathbf{x} \oplus \mathbf{a}$ (bit-wise xor operation), where a is a fixed n-bit string. Since f_a is invertible, its unitary can be directly written as $U_{\mathbf{a}} : U_{\mathbf{a}}(|\mathbf{x}\rangle) = |\mathbf{x} \oplus \mathbf{a}\rangle$. Show that the two following quantum circuits, on the same input state, output the same quantum state:

Exercise 6: No-cloning Theorem

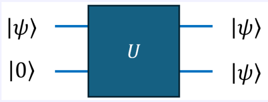
Let $|\psi\rangle$ be a single-qubit state. Here we want to "copy" $|\psi\rangle$. Suppose that there exists a unitary U that can accomplish this task:



Show by contradiction that such U does not exist (hint: Analyze the case where $|\psi\rangle$ is in superposition). Then explain why the controlled-NOT gate cannot copy quantum states.

Exercise 7: No-cloning Theorem

Let $|\psi\rangle$ be a single-qubit state. Here we want to "copy" $|\psi\rangle$. Suppose that there exists a unitary U that can accomplish this task:



Show by contradiction that such U does not exist (hint: Analyze the case where $|\psi\rangle$ is in superposition). Then explain why the controlled-NOT gate cannot copy quantum states.