Quantum Computing

- Week 6 (May 19-20, 2026)

- Topics:
  - Deutsch-Jozsa Problem
  - Entanglement
  - Pure states and mixed states

The Deutsch-Jozsa Problem

- Constant-vs-balanced problem
- Let $f: \{0,1\}^n \rightarrow \{0,1\}$ be a bit function such that it is in either two cases:
  - f is a *constant* function: $\forall \boldsymbol{x} \in \{0,1\}^n, f(\boldsymbol{x})$ is always a constant (0 or 1)
  - f is a *balanced* function: $\sum_{x \in \{0,1\}^n} f(\boldsymbol{x}) = \boldsymbol{2^{n-1}}$ (i.e., outputs 0 for half the inputs, and 1 for the other half)

- To decide whether f is constant or balanced, **how many times** must we evaluate f?

Classical Computer

Worst-case: 2^{n-1} (Check half of inputs)

Probabilistic algorithm (Check l inputs):

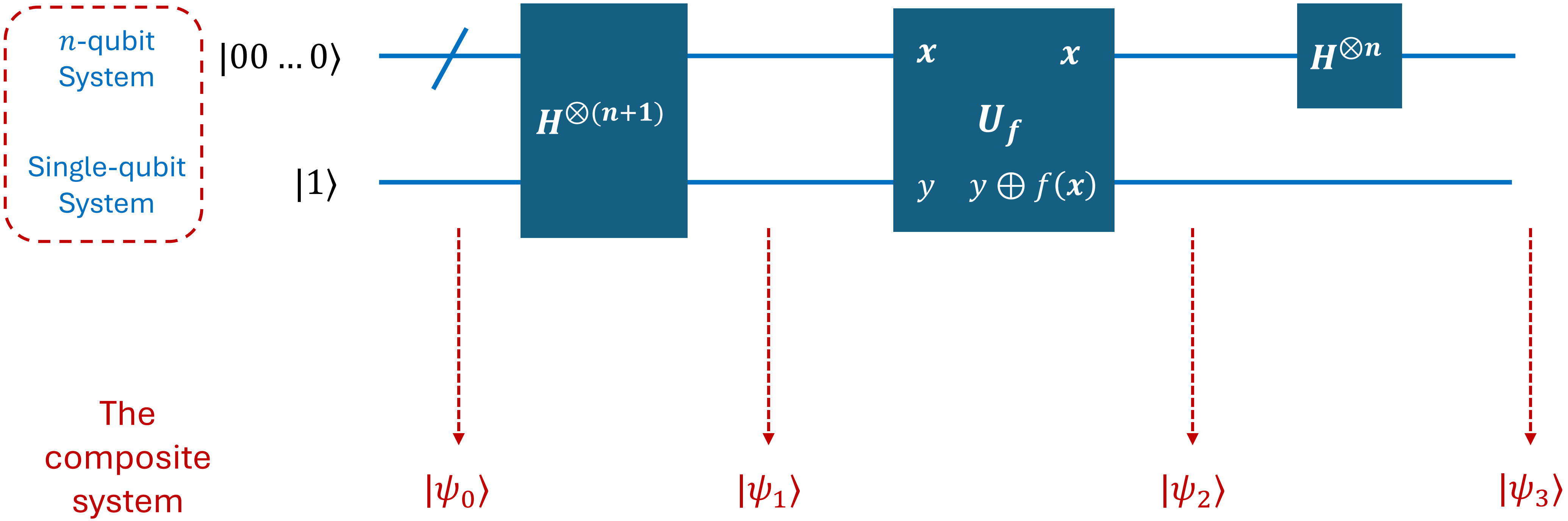
$l \ll 2^{n-1}$ times,

with a failure rate of $\frac{1}{2^l}$

Can we do better?

The Deutsch-Jozsa Algorithm

- Let $f: \{0,1\}^n \rightarrow \{0,1\}$ be a bit function...
- (Do it on the board)



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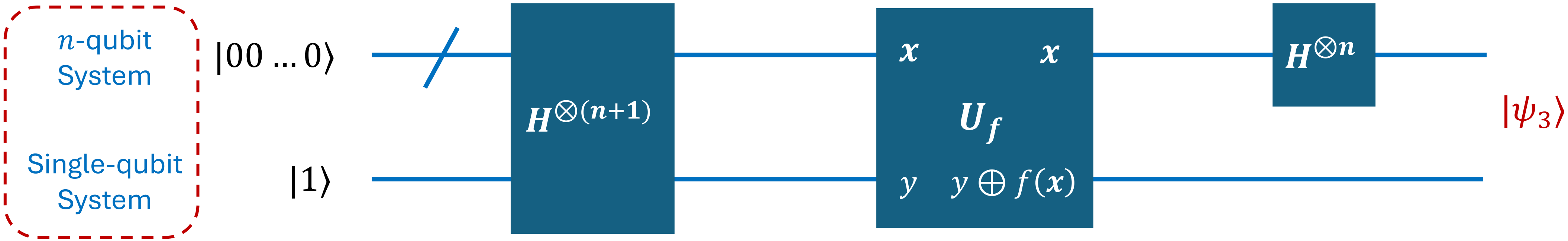

The composite system

$$|\psi_3\rangle = \left(\sum_{z \in \{0,1\}^n} \sum_{x \in \{0,1\}^n} \frac{(-1)^{x^T z + f(x)} |z\rangle}{2^n} \right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

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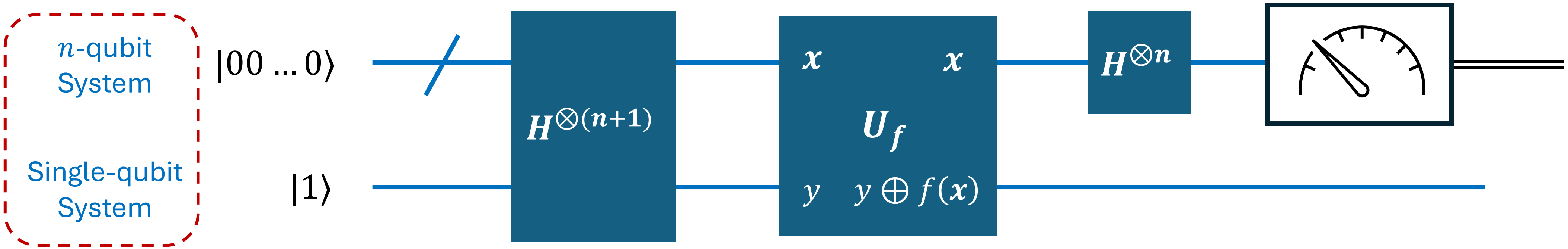

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Measure the first n systems ?

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$\sum_{\boldsymbol{x}\in\{0,1\}^n}\frac{(-1)^{f(\boldsymbol{x})}}{2^n} = 1$ or -1

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$$\sum_{x \in \{0,1\}^n} \frac{(-1)^{x^T z + f(x)}}{2^n} \neq 0$$

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Quantum Computer:

Evaluate **once**,

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Quantum Supremacy

The Deutsch-Jozsa Problem

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Quantum Supremacy

But wait…What's **the practical application** of the Deutsch-Jozsa problem?

Quantum-Classical Separation Problems

- The Deutsch–Jozsa problem has no known practical application
- It is an early example of quantum supremacy, illustrating (or suggesting) the theoretical **separation between quantum and classical computation** (e.g., BPP vs BQP)

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- Similar quantum-classical separation problems:
  - Simon's problem (exercise or homework, TBD)

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- Separation problems that have **practical use**:
  - Hidden subgroup problem (Discrete logarithm, Factoring): Shor's algorithm
- No quantum supremacy, but **quantum acceleration**
  - Unstructured search problem: Grover's search algorithm

Postulates of Quantum Computing

- Postulate 1: State space
- Postulate 2: Evolution and unitary transformation
- Postulate 3: Quantum Measurement
    - Projective measurement
- Postulate 4: Composite system

Postulates of Quantum Computing

- Postulate 1: State space **(isolated systems)**
- Postulate 2: Evolution and unitary transformation **(closed systems)**
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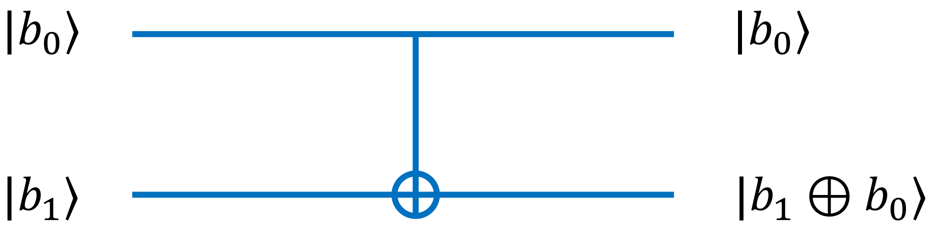
Controlled NOT Gate

- CNOT: If $b_0 = 0$, output b_1; Else, output $1 \oplus b_1$ (i.e., flip b_1 if $b_0 = 1$)



- $|00\rangle \rightarrow |00\rangle$
- $|01\rangle \rightarrow |01\rangle$
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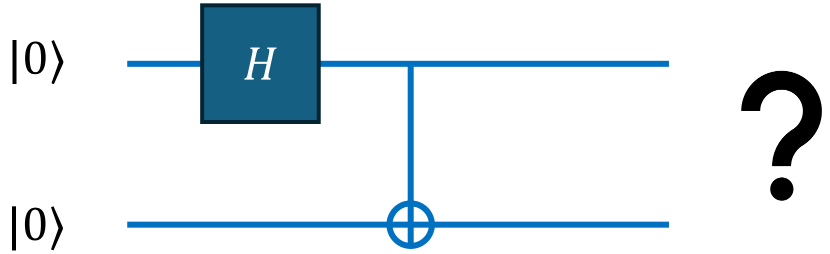

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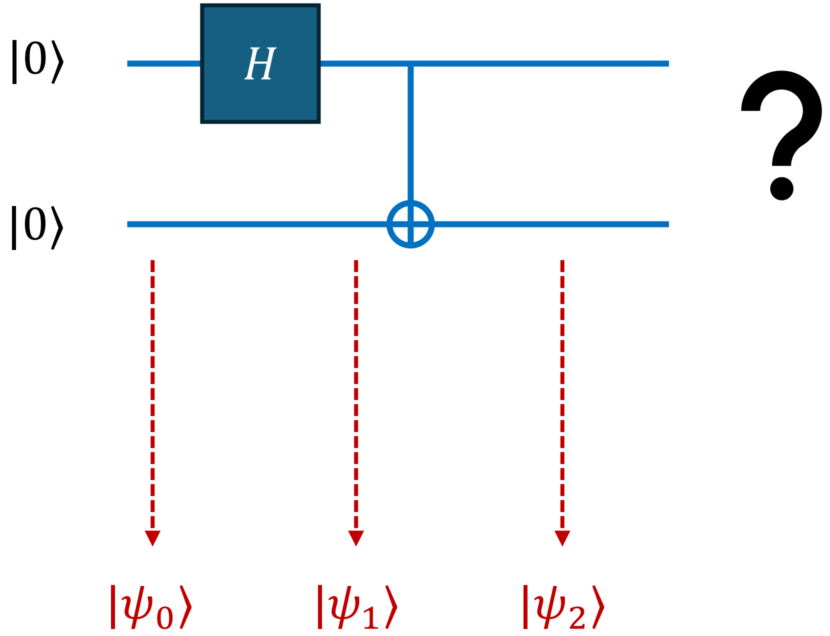

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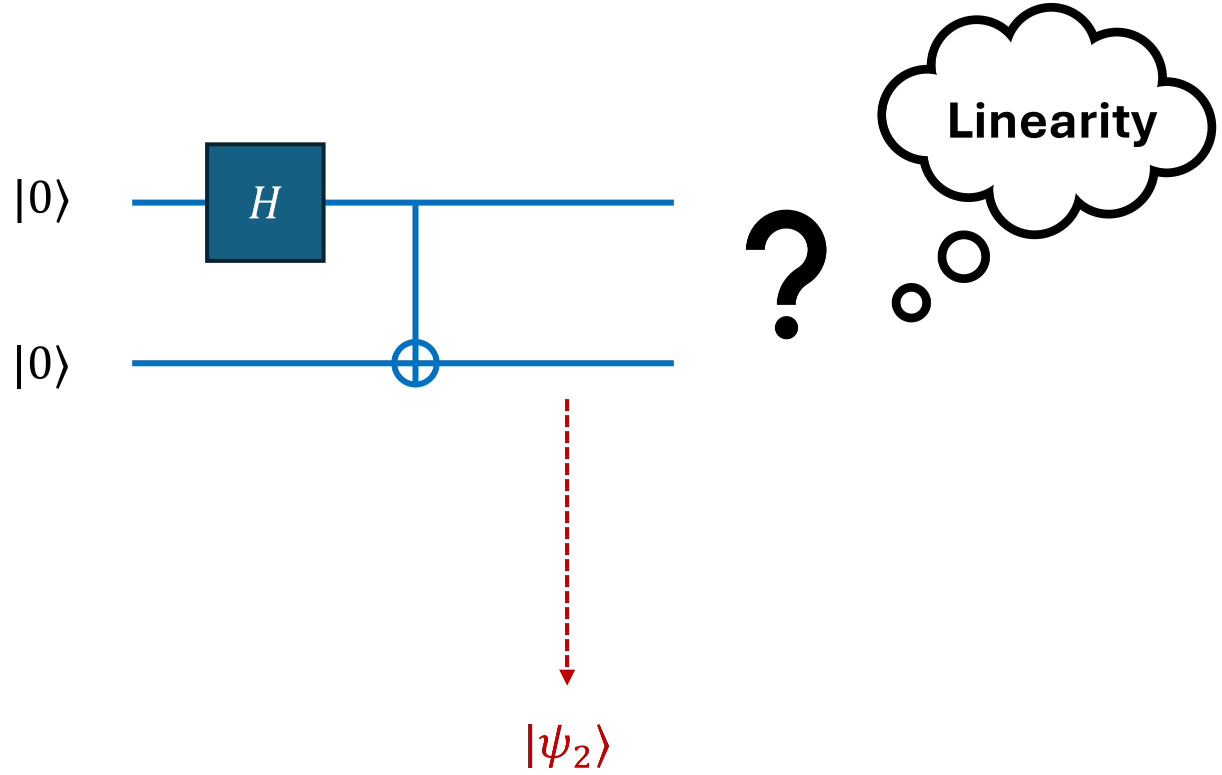

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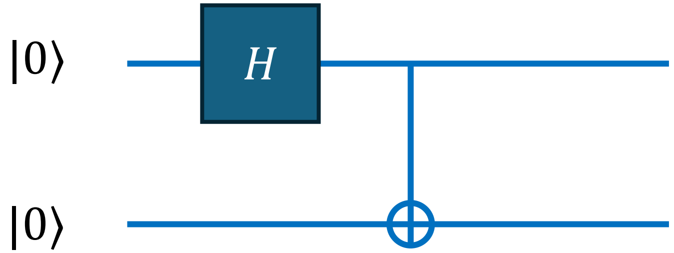

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

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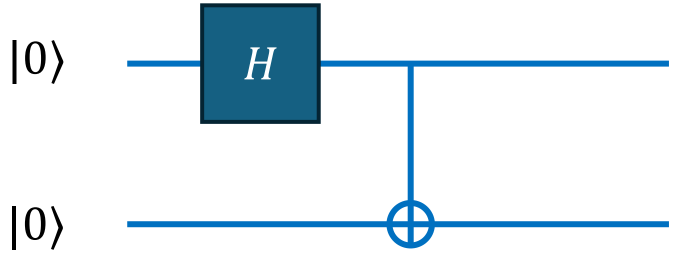

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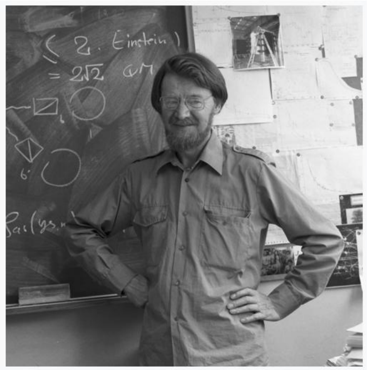
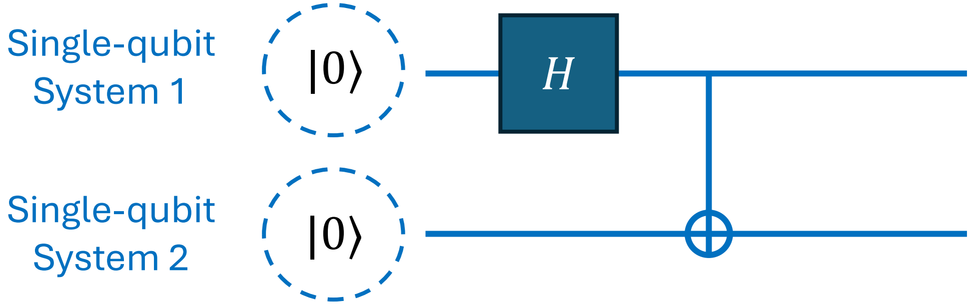


John Stewart Bell
(source: Wikipedia)

Bell state:
Impossible to be split into
a tensor product of two states
$|\varphi_1\rangle \otimes |\varphi_2\rangle$

Quantum Entanglement



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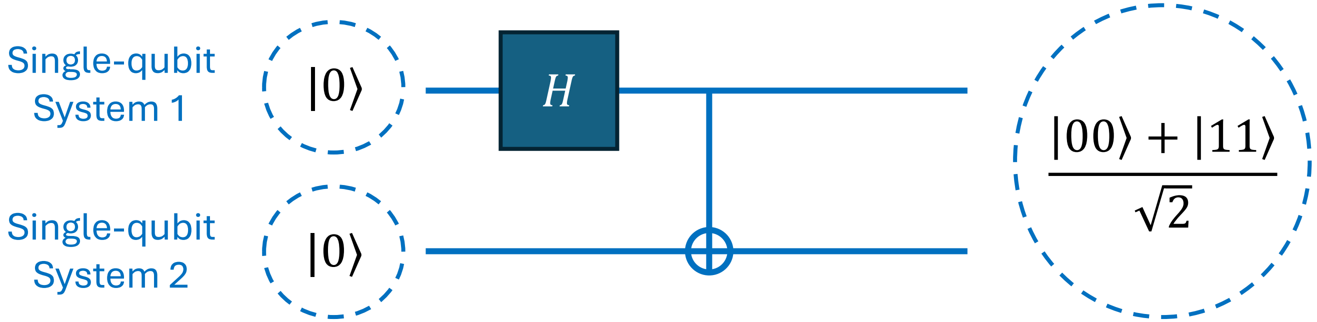
Now the two systems are entangled

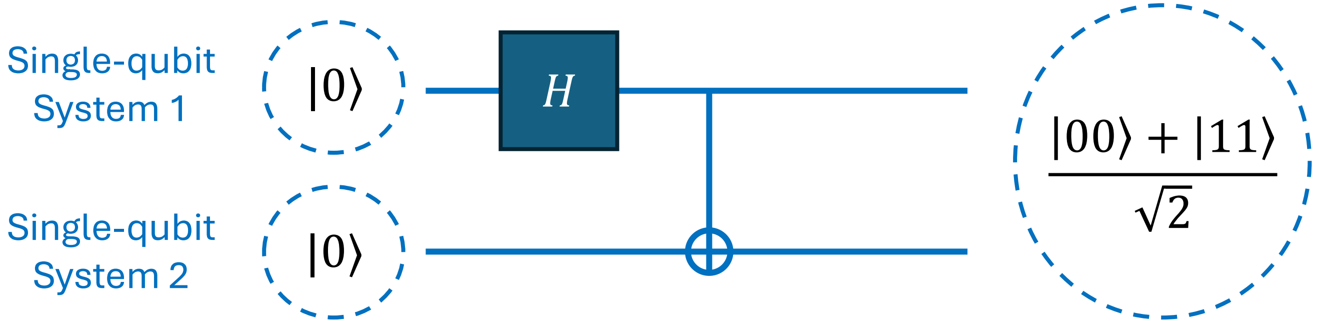
Quantum Entanglement



Now the two systems are entangled

Pure state: Can be described by a state vector
Mixed state: Can**not** ...

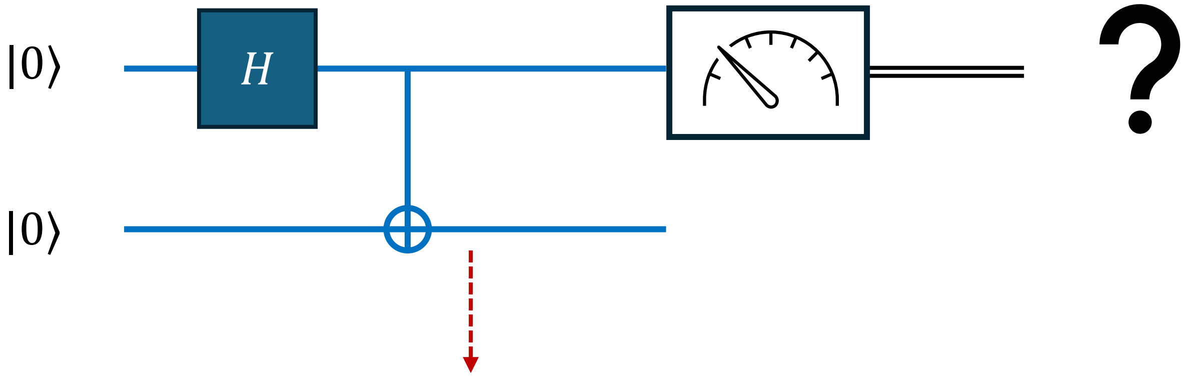
Quantum Entanglement



Small Exercise: (pure or mixed)

1. The initial state of system 1 is ____.
2. The states of system 1 and 2 (after H and CNOT) are ____.
3. The state of the total system (after H and CNOT) is ____.

Partial Measurement on Entangled States



$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Partial Measurement on Entangled States

- Formalizing Partial Measurement
  - Let's focus on the computational basis
  - General measurement: $\{M_m\}_m \rightarrow \{M_m \otimes I\}_m$
  - Projective measurement: $\boldsymbol{M} = \sum_m m\boldsymbol{P}_m \rightarrow \boldsymbol{M} \otimes \boldsymbol{I} = (\sum_m m\boldsymbol{P}_m) \otimes \boldsymbol{I} = \sum_m m(\boldsymbol{P}_m \otimes \boldsymbol{I})$

Partial Measurement on Entangled States

Spectral decomposition (simplified):

Any Hermitian operator $\boldsymbol{M}$ (i.e., $\boldsymbol{M} = \boldsymbol{M}^{\dagger}$) can be written as:

$$\boldsymbol{M} = \sum_{\lambda} \lambda \boldsymbol{P}_{\lambda}$$

- λ represents an eigenvalue of $\boldsymbol{M}$ (also indexes the projectors)
- $\boldsymbol{P}_{\lambda}$ represents the projector onto the λ eigenspace, $\boldsymbol{P}_{\lambda}^{2} = \boldsymbol{P}_{\lambda}$
- $\boldsymbol{P}_{\lambda}$ itself is also Hermitian, i.e., $\boldsymbol{P}_{\lambda} = \boldsymbol{P}_{\lambda}^{\dagger}$
- $\sum_{\lambda} \boldsymbol{P}_{\lambda} = I$
- $\boldsymbol{P}_{\lambda}^{\dagger} \boldsymbol{P}_{\lambda'} = \delta_{\lambda,\lambda'} (= 1 \text{ if } \lambda = \lambda', \text{otherwise} = 0\ )$

Partial Measurement on Entangled States

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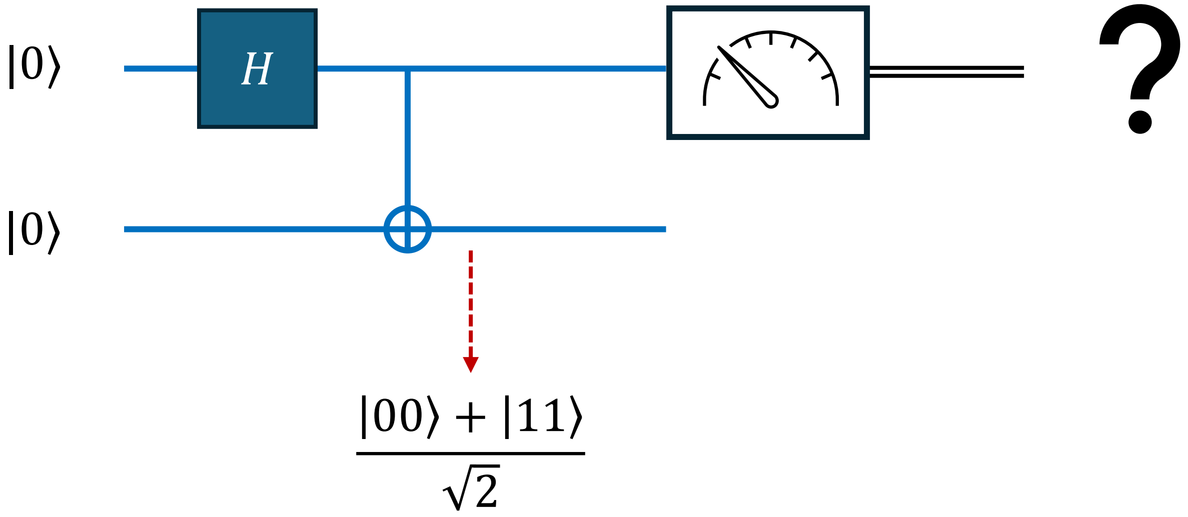
  - Projective measurement: $\boldsymbol{M} = \sum_m m\boldsymbol{P}_m \ \rightarrow \ \boldsymbol{M} \otimes \boldsymbol{I} = (\sum_m m\boldsymbol{P}_m) \otimes \boldsymbol{I} = \sum_m m(\boldsymbol{P}_m \otimes \boldsymbol{I})$

- Important notes:
  - $\{M_m \otimes I\}_m$ still satisfies the completeness equation
  - $\boldsymbol{M} \otimes \boldsymbol{I}$ is still an observable

- Example (Exercise):
  - Partial measurement on the state $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

Partial Measurement on Entangled States



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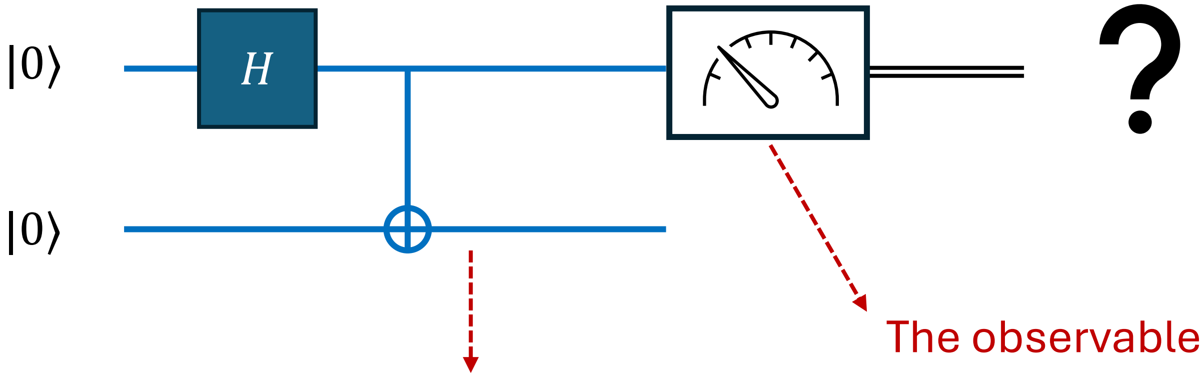

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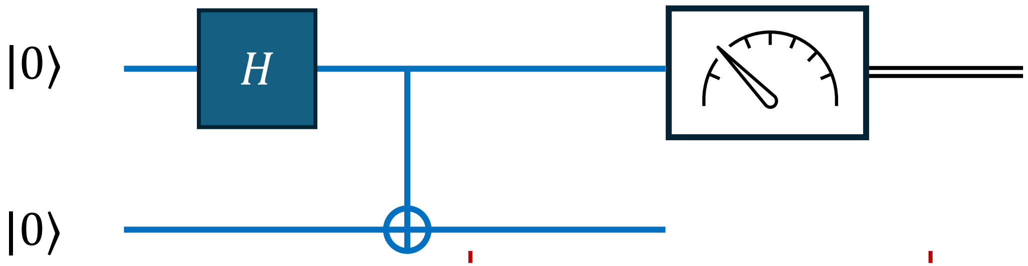
The observable of the measurement is

$$\boldsymbol{M} = 0 \cdot |0\rangle\langle 0| + 1 \cdot |1\rangle\langle 1|$$

Partial Measurement on Entangled States



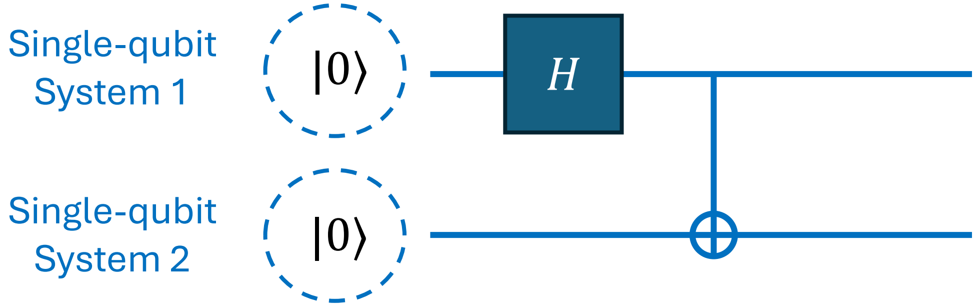
$$\begin{cases} 0. & \text{with prob. } ½ \\ 1. & \text{with prob. } ½ \end{cases}$$

For entangled states, **partial measurement leads to global collapse**

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

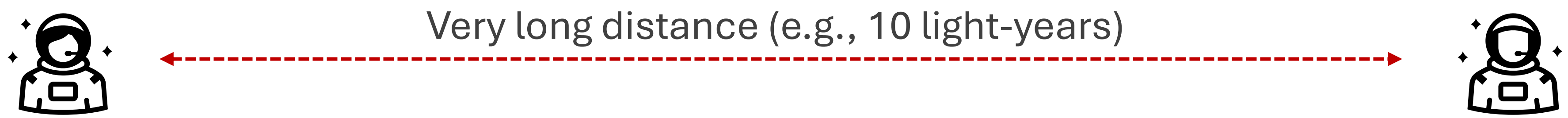
State after measurement $\begin{cases} |00\rangle & \text{if the measurement outome is } 0 \\ |11\rangle & \text{if the measurement outome is } 1 \end{cases}$

Action at a Distance (Fernwirkung)



$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Action at a Distance

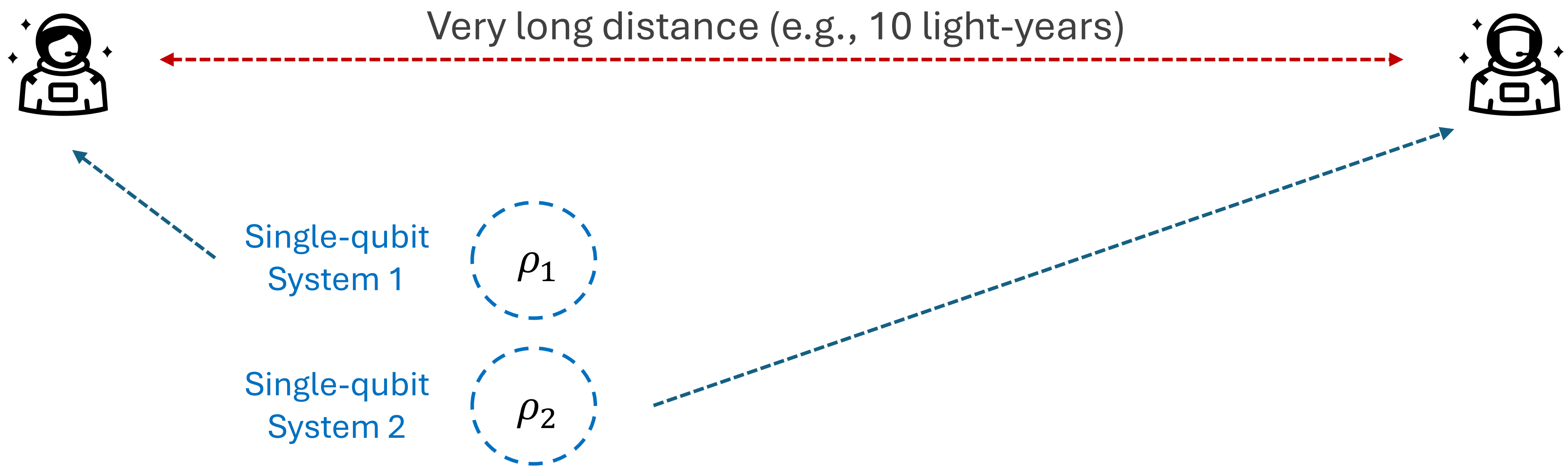


Single-qubit System 1 — ρ_1

Single-qubit System 2 — ρ_2

Total state: $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Action at a Distance



Total state: $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$

Action at a Distance

Very long distance (e.g., 10 light-years)

Single-qubit System 1

ρ_1

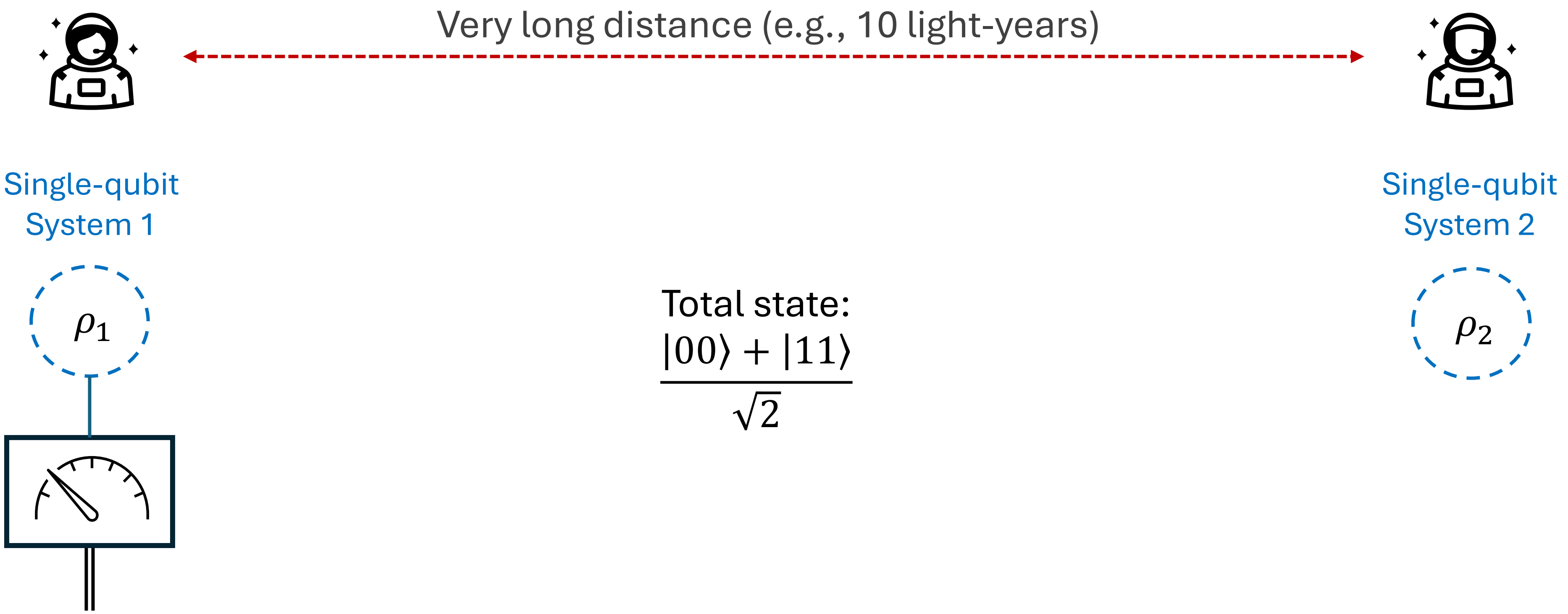
Single-qubit System 2

ρ_2

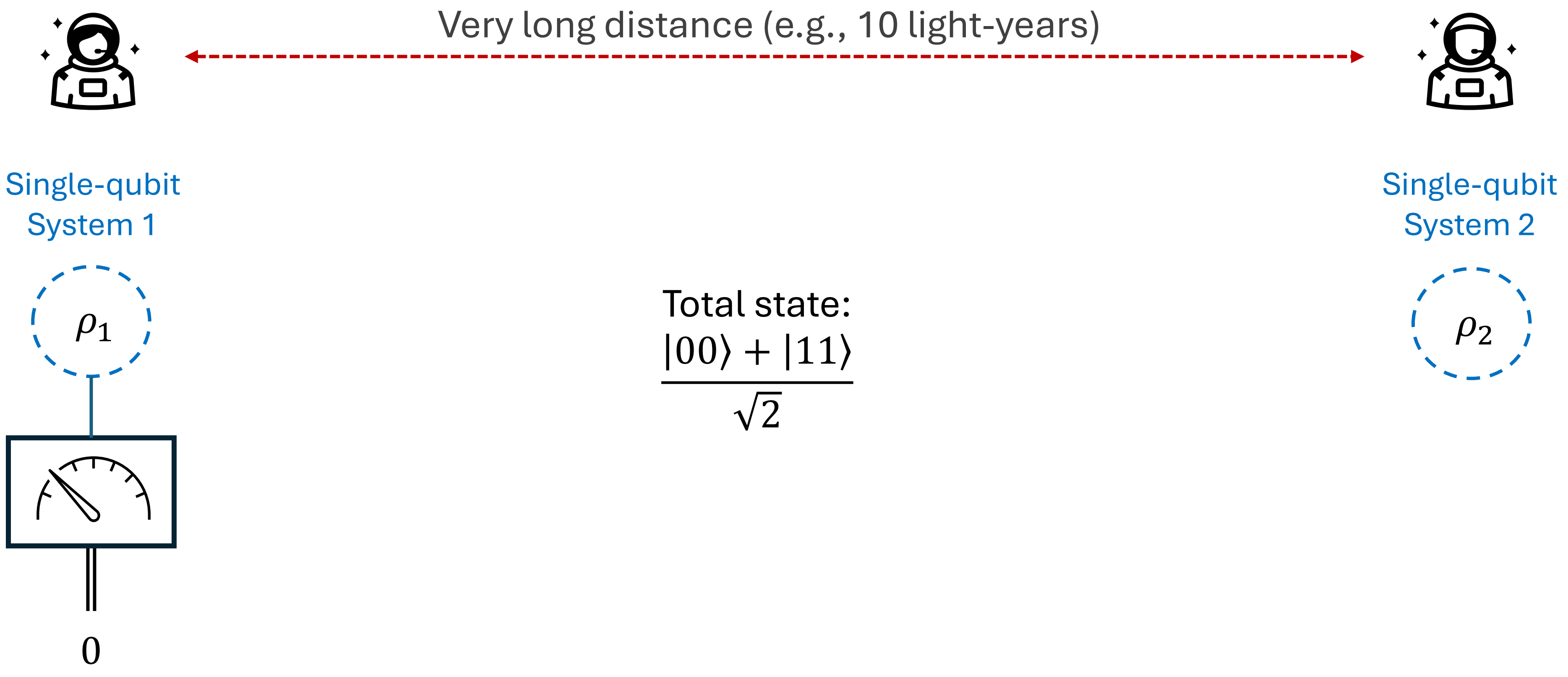
Total state:

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

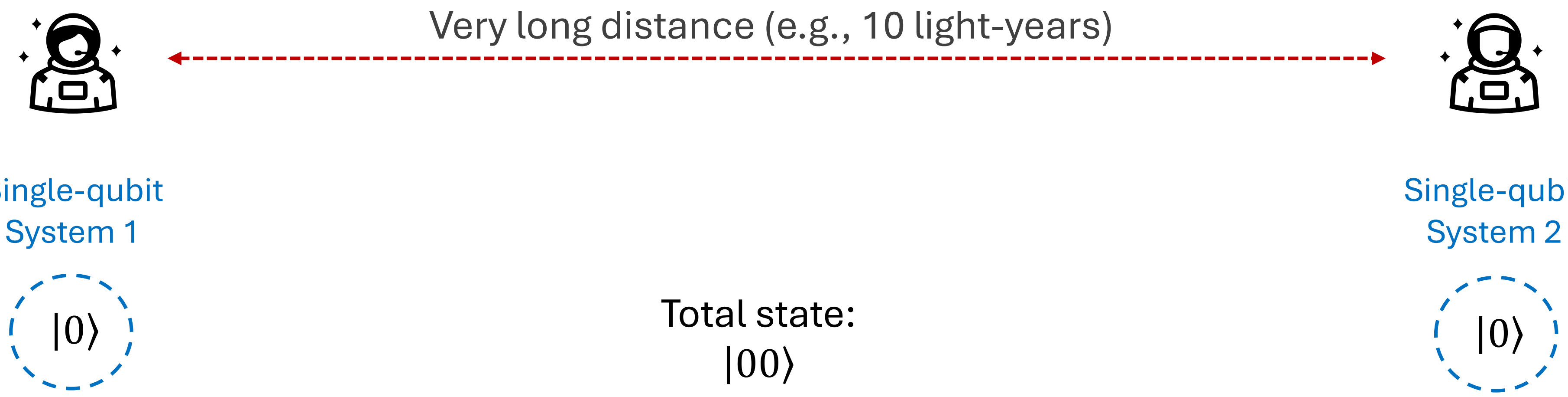
Action at a Distance



Action at a Distance

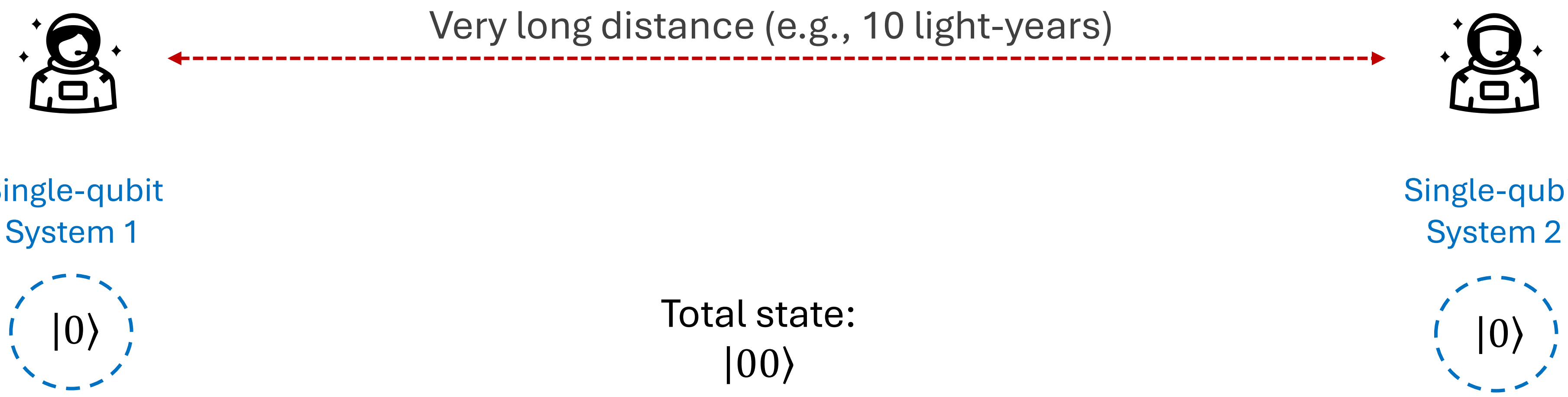


Action at a Distance



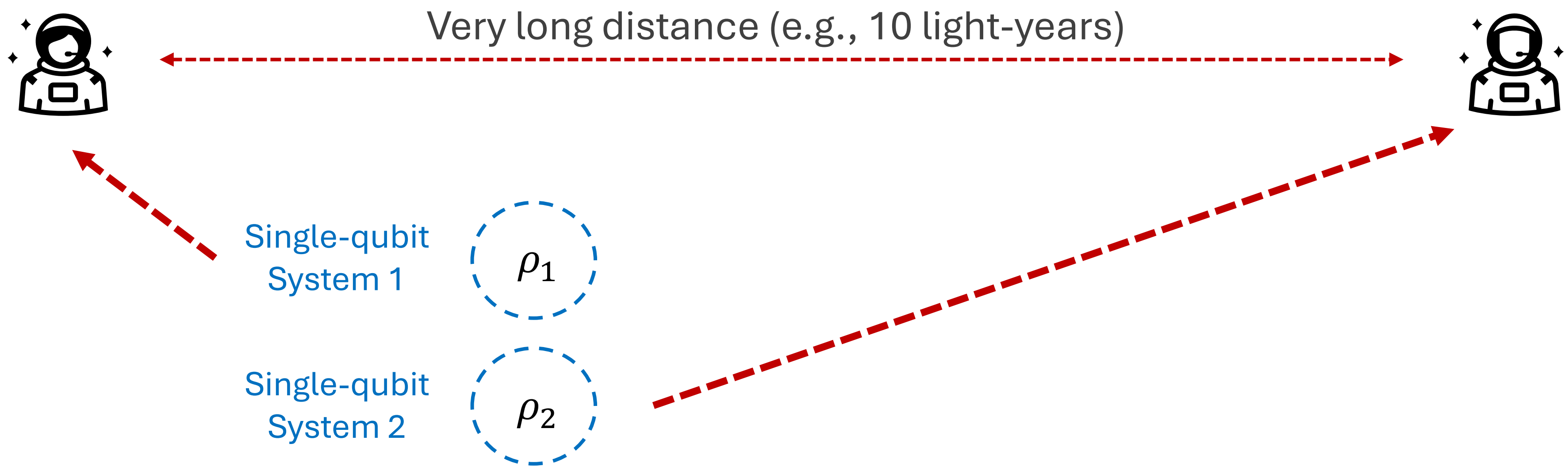
- “spukhafte Fernwirkung”

Action at a Distance



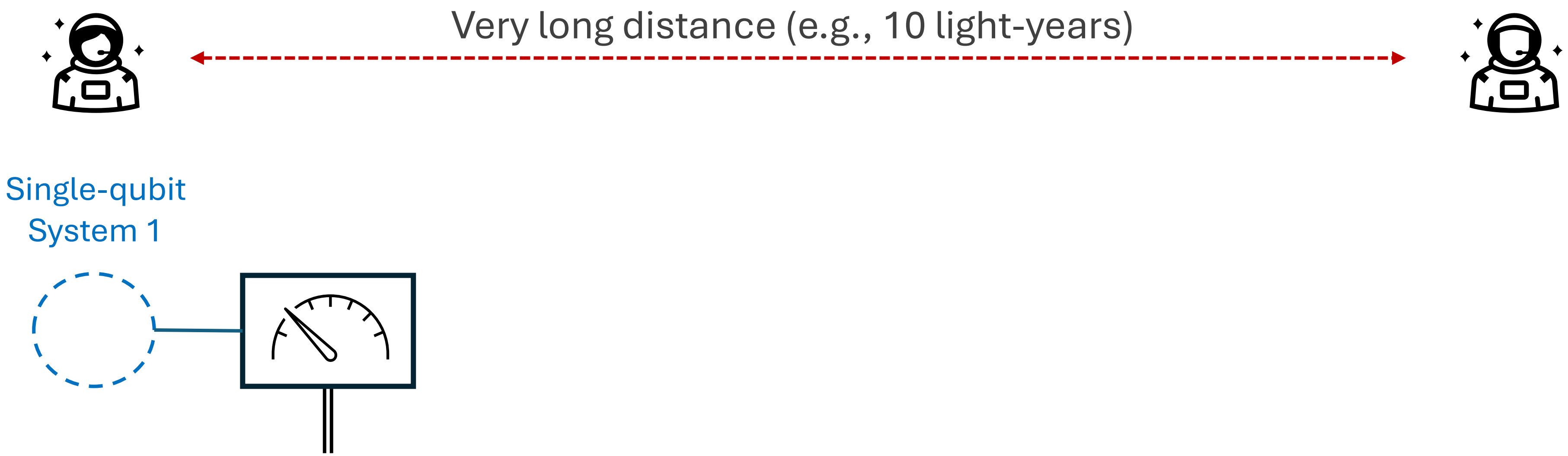
- “spukhafte Fernwirkung”
- A quick question: Is it a faster-than-light **communication**?

Action at a Distance



- “spukhafte Fernwirkung”
- A quick question: Is it a faster-than-light **communication**? **No**
  - They require some pre-shared entangled states

Action at a Distance



- “spukhafte Fernwirkung”
- A quick question: Is it a faster-than-light **communication**? **No**
  - They require some pre-shared entangled states
  - We cannot control the measurement output (i.e., random bit)
  - The receiver has no idea if its system has been measured

Next Week

- Superdense coding (...one qubit “encodes” two classical bits...)
- Quantum teleportation (...transfer one qubit via sending two classical bits...)